Important questions include 'explain', 'why', or 'how' a child reached a particular conclusion. This informs the teacher about what the child knows. A focus on strengths helps a teacher formulate what the next step for a particular child should be.

There are two observations, which we think are significant for the practice of education in general. These observations – and the prior work we have reported – show the payoff of a problem-solving approach to learning. Rather than being shown a bunch of patterns and asked to remember and produce them at test time, children responded to the challenge *create some patterns*.

Learning is provoked adaptation

Throughout the process, the children were asked to think about the problem with which they were involved. They reflected on similarities between patterns, used patterns to demonstrate an organisation to their thinking, and were often exhaustive in their work. They did this because they were engaged by the problem; they were given opportunities to be creative; the social component was supportive and safe, even when there was disagreement. They thought about, questioned, and articulated their thinking.

And, quite different from the research approach so commonplace in today's education, which investigates the effect of the treatment on the organism (i.e., 'What changes are brought about by this or that brand of breakfast cereal, this television commercial or this curriculum?'), we are interested in *the effect of the organism on the treatment*. That is, what do children make of things? How do they act on, construct, represent ('to bring to mind by description,') the world?

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Notes

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FROM MATHEMATICS TO MATHEMATICS-WITH-ICT

Jay Timotheus continues to expand his ideas with specific reference to the working classroom. [Part 1 was published in *MT212*]

In the first part to this article (***MT212***), I described three stages that I believe are effective starting points for translating familiar classroom activities into mathematics-with-ICT lessons that exploit the power of variation to focus student attention.

The three stages are:

Stage one – The teacher identifies significant aspects in the mathematical situation that they want the students to notice.
What is it about the situation that is worth becoming aware of? What are the intended mathematical connections? What is it that the teacher would like the students to see?

Stage two – The teacher identifies properties of the mathematical situation that are (a) variable and (b) will draw attention, when varied, to aspects that they want the students to notice. The properties being varied may be fundamental constraints of the mathematical situation that the teacher may never have considered varying before.

Stage three – The teacher identifies mathematics software that enables him or her to make the variation explicit and plans an activity around this. The questions to ask of students when the variation takes place are carefully thought through in order to draw attention to the precise aspects that the teacher wants the students to become aware of.

In this article I describe two such activities to illustrate this approach.

Constructing triangles of given lengths – 1

The teacher identifies significant aspects in the mathematical situation that they want the students to notice.

I once heard a teacher giving instructions to their class, explaining how to draw a triangle with three given sides. "The first thing you have to do is find the longest side and draw that with your ruler," the teacher began. And so began a sequence of instructions that, if followed would indeed produce triangles with the given lengths. However, for me, a number of questions were raised. Why draw the longest side first? Why use compasses? When thinking about the drawing of triangles of given lengths it is easy to consider this 'algorithmic' construction as no more than that – a sequence of steps to be followed in order to achieve the required result. Presenting students with such an algorithm in a lesson may lead to exercise books full of correctly constructed triangles, but also leaves students at the end of the lesson with the requirement to remember the steps correctly without having had the opportunity to engage with the underlying mathematics and gain a sense-of-why these steps work. It is certainly not intuitive to students attempting to draw a shape with straight sides that compasses may be helpful. I believe that the students need to be given the opportunity to see that constructing a triangle of given lengths relies on the ability to create loci by rotating fixed lengths. This, for me, is where the mathematics lies and we can take this as our starting point for the creation of a mathematics-with-ICT activity involving variation.

Constructing triangles of given lengths – 2

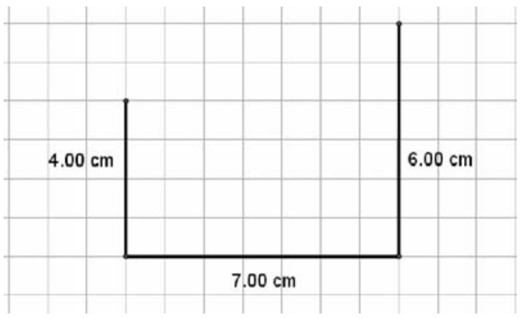
Identify some aspect in the mathematics of the situation that is (a) variable and (b) will draw attention to what it is you want your students to notice when it is varied.

Having identified that I want to focus my students' attention on the ability to rotate given lengths in order to construct a triangle, I am drawn to the image of a line segment having two 'arms' that swing around the endpoints and meet in two possible places. The vary-able aspect here is the orientation of the 'arms'. The moment when the moving arms join together to make a triangle is significant as it will suddenly draw the observer to recognise a familiar shape.

Constructing triangles of given lengths – 3

The teacher identifies mathematics software that enables him or her to make the variation explicit and plans an activity around this.

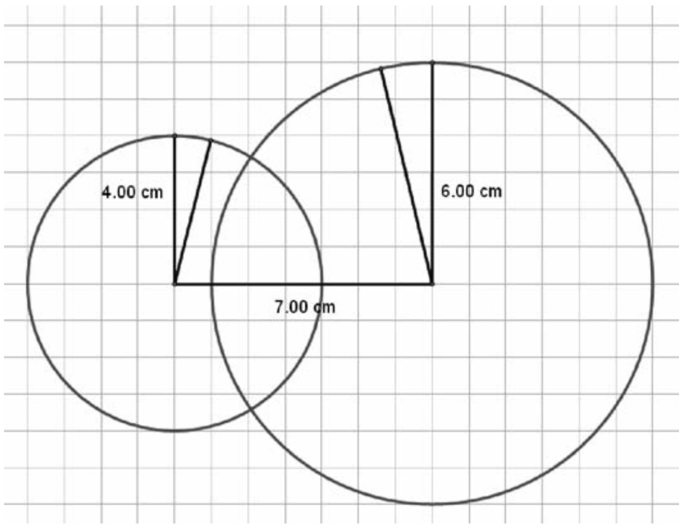
There are a number of computer software applications in which geometric figures can be constructed and manipulated. In this example I use *Geometers' Sketchpad*. However, when working with students on constructing triangles I often begin by giving my students three side lengths (for example 7cm, 6cm and 4cm) and then ask them to draw the triangle described in their exercise books. The next ten minutes or so are spent with the students busily drawing triangles with their pencils and rulers, and finding, in most cases, that it is quite difficult to achieve three sides all with the required lengths. There is soon a sense in the room that 'this is actually harder than we thought.' It is at this point that a sense of 'need' has been created to find a better way.



This figure is then constructed by drawing three line segments that 'snap' to the grid..

The teacher can then use the 4cm and 6cm lines as radii and construct two circles as shown. Two further radii are then drawn which are rotated by dragging until a triangle is formed.

See: www.atm.org.uk/mt213.



There is something about the construction of this image that is powerful in a way that cannot be conveyed on the printed page. As the two radii are dragged towards the intersection of the circles, the triangle is suddenly apparent and I have found that this always provokes an excited response from many students in the room. Suddenly they see that these (round) circles have in fact allowed us to rotate the lengths and to make the very same triangle (with straight sides!) that seemed so difficult to construct earlier with a (straight) ruler and a pencil. I find it helpful to try and channel the generated excitement at this point by asking further questions.

- Is this the triangle we wanted?
- How do we know?
- Is there another triangle that will also have the same side lengths?
- Why did we draw circles?
- . . . ?

The mathematics-with-ICT lesson presented here differs in approach from the more traditional algorithmic approach to teaching this construction. The ICT part of the lesson starts 'in the wrong place'. Rather than beginning with one length and trying to construct the other two (segment – arc – arc – segment – segment), it begins with three lengths and tries to manipulate these three lengths into the right place (segment – segment – segment – circle – circle – rotate segments into place). There is often a difference between the order in which things fit into place in mathematics and the order in which they can be best understood. Long before computers were being used in schools, Pólya wrote, *Mathematics has two faces; it is the rigorous science of Euclid but it is also something else. Mathematics presented in the Euclidean way appears a systematic, deductive science; but Mathematics in the making appears an experimental, inductive science.* (Pólya, 1945, p. xxxiii)

Adding fractions visually – 1

The teacher identifies significant aspects in the mathematical situation that they want the students to notice. I am sure that I am not alone in my experiences of working with students that find fractions difficult to understand. Recognising that $\frac{2}{5}$ has the same value as $\frac{6}{15}$ is not obvious to many students. Recognising that it is useful to find equivalent fractions in order to add $\frac{2}{5}$ and $\frac{1}{3}$ is a similarly elusive idea. How then can an ICT lesson involving variation be created to give students the opportunity to notice the equivalence of fractions and its importance in addition?

Adding fractions visually – 2

The teacher identifies properties of the mathematical situation that are (a) variable and (b) will draw attention, when varied, to aspects that they want the students to actively notice.

Let us consider a rectangle that has been split into two parts. $\frac{2}{5}$ is shaded, $\frac{3}{5}$ is not shaded. There are no further divisions. Some students may be able to 'guess' that $\frac{2}{5}$ has been shaded – but it is not obvious.

If a line is then drawn dividing the rectangle into two equal parts it can be seen that the shaded part is not $\frac{1}{2}$.

If that line is erased and replaced with lines dividing the rectangle into three equal parts it can be seen that the shaded part is neither $\frac{1}{3}$ nor $\frac{2}{3}$ (nor $\frac{3}{3}$).

This can continue until the rectangle has been divided into five equal parts. This time it will immediately become notice-able that 2 out of 5 equal parts are shaded and that this fraction can be described as $\frac{2}{5}$.

The process can continue further, dividing the rectangle into more equal parts. When the rectangle is divided into ten equal parts, it will become apparent that 4 out of 10 equal parts have been shaded, enabling the equivalence between $\frac{2}{5}$ and $\frac{4}{10}$ to be actively noticed.

Adding fractions visually – 3

The teacher identifies mathematics software that enables him or her to make the variation explicit and plans an activity around this.

Having identified that the number of divisors of the rectangle as the variable property, I chose the *TI-Nspire* software in order to construct the file that is shown here, which is also available from the ATM website, www.atm.org.uk/mt213.

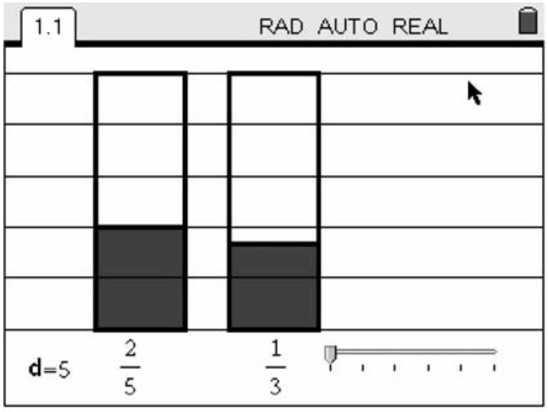


figure 1

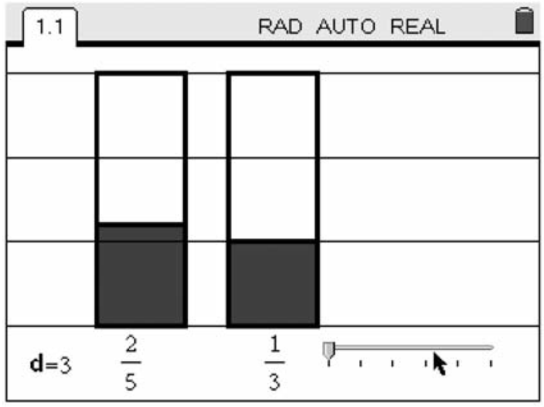


figure 2

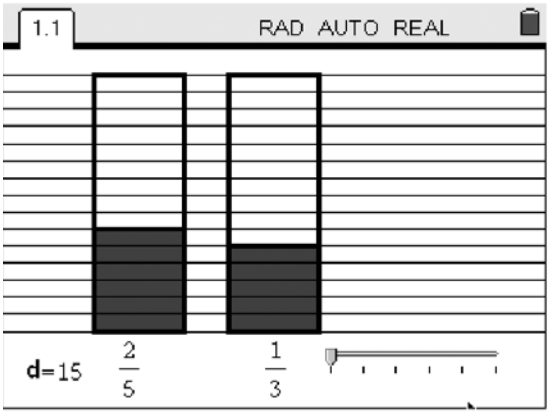


figure 3

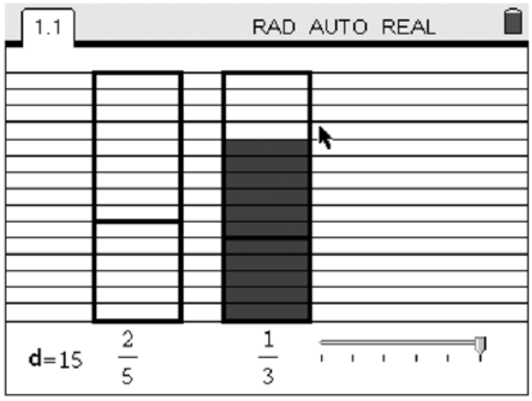


figure 4

The screen images above show two fractions to be added. Varying the value of *d* changes the number of equal divisors that the fraction has been split into. When working with students using this program, the teacher can ask students to choose for themselves how many divisors to split the shape into. They will be able to see for themselves the values of *d* that are helpful or unhelpful and hopefully begin to see why. (The program also offers a slider that transfers the area shaded from the first rectangle into the second rectangle, as well as allowing the student to change the two fractions, see *figure 4*.)

Summary

In these two examples I have tried to illustrate how variation can be used to make mathematical properties explicit and offered a three stage process which can be used to create lessons involving variation using ICT. Computers allow students to see instantly the results of changes and this is a powerful tool for focusing attention on mathematical properties. It is often a challenge for us as teachers to identify the 'significant aspects' of mathematical situations and then go on to find ways of using variation to make the mathematics more noticeable. I hope that the framework presented here encourages more of us to find ways of doing so.

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Reference

Pólya, G., (1945), *How to Solve It*, Princeton University Press