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A PRACTITIONER MODEL OF THE USE OF COMPUTER-BASED TOOLS AND RESOURCES TO SUPPORT MATHEMATICS TEACHING AND LEARNING

ABSTRACT. This study analyses the pedagogical ideas underpinning teachers' accounts of the successful use of computer-based tools and resources to support the teaching and learning of mathematics. These accounts were elicited through group interviews with the mathematics departments in seven English secondary schools, conducted in the first half of 2000. The central themes are organised to form a pedagogical model, capable of informing use of such technologies in classroom teaching, and of generating theoretical conjectures for future research. The thematic components of the model are summarised and related to prior research into computer use in mainstream schooling.

KEY WORDS: computer uses in education, educational technology, England, mathematics teaching and learning, pedagogical models, research report, secondary schools, teacher thinking, teaching strategies

RESEARCH INTO COMPUTER USE IN SCHOOL MATHEMATICS

The incidence of computer use in mainstream school mathematics

Although there has long been interest in the potential of computer-based tools and resources in school mathematics, their use has only recently become more established in mainstream practice. An indication of the extent of such use is provided by the recent international surveys conducted as part of the original and repeated implementations of the Third International Mathematics and Science Study [TIMMS] (Beaton, Mullis, Martin, Gonzalez, Kelly and Smith, 1996: 168; Mullis, Martin, Gonzalez, Gregory, Garden, O'Connor, Chrostowski and Smith, 2000: 217–8). Averaged across the 23 educational systems participating in both surveys, the proportion of eighth-grade students reporting some degree of computer use in their mathematics classes rose, between 1995 and 1999, from 17 per cent to 21 per cent, while the proportion reporting such use as occurring more than 'once in a while' remained steady at 5 per cent.

In the first survey, England – at 55 per cent – was the system with easily the largest proportion of students reporting computer use, followed by the United States – at 31 per cent, and both these figures were considerably

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higher than the median across systems – at 11 per cent. In the second survey, England was joined by Singapore, the system showing by far the most marked increase over the intervening period – rising by 44 percentage points. In only three other systems – Canada, Hong Kong, and Korea – was there an increase of 10 percentage points or more, with the median change across systems being a rise of 2 percentage points. Typically, then, computer use remains low, and its growth slow.

Trends in research on computer use in school mathematics

Against this background, it is not surprising that studies of computer use in school mathematics have largely examined explicitly innovative situations, usually linked to development projects of some type (as is readily illustrated by scanning the studies previously published in *Educational Studies in Mathematics).* Equally, the focus of these studies has been predominantly on student cognition and machine interaction. Nevertheless, some studies have investigated the pedagogical perspectives and practices of teachers participating in development projects, curriculum reforms and in-service courses promoting use of particular computer-based resources and tools: for example, use of diverse interactive video materials to support a range of mathematical tasks, at secondary level in England (Phillips, Pead and Gillespie, 1995); use of Logo programming to mediate mathematical problem solving and investigation, at primary level in Portugal (Moreira and Noss, 1995); and use of programming, data handling and graphing tools to operationalise basic ideas and processes of informatics, at secondary level in Italy (Bottino and Furinghetti, 1996).

Recent studies of computer use in mainstream practice

More recently – and very predominantly in the United States – studies have started to examine the evolution of computer use beyond situations of innovation, in circumstances where it has become more ordinary than innovatory, more established in operation than consciously under development. A nation-wide questionnaire survey of teachers in the United States found that the predominant use of computers in teaching mathematics, both at upper elementary and secondary levels, was to provide practice of skills (Becker, Ravitz and Wong, 1999: 13). Correspondingly, mathematics teachers as a group were found to be relatively strongly oriented towards a 'transmission' view of teaching as opposed to a 'constructivist' one (Ravitz, Becker and Wong, 2000: 17). Comparing the profile of the most 'constructivist' mathematics teachers with that of their peers, the former made more use of word-processing and less of skill-practice games; levels of use of simulations/exploratory environments and of spreadsheets/databases were similar; and none of these were employed extensively by either group (Becker, 2000: 17, 20).

A state-wide questionnaire survey (Niederhauser and Stoddart, 2001) examined the types of software that elementary school teachers used in their teaching, and their ratings of the effectiveness of computer use for different types of instructional purpose. Although the study was conceptualised in terms of a contrast between 'didactic' and 'constructivist' pedagogy, around half the respondents reported using both 'skill-focused' software (such as drill-and-practice) and 'open-ended' software (such as mathematical games, spreadsheets and Logo). Similarly, analysis identified teachers who viewed computer use as effective both in providing drill, practice and reinforcement, and in supporting analytic, creative and independent thinking. The study speculates that these teachers may have been "sophisticated users who chose different types of software to meet specific educational goals", or that they "may simply have used all of the different types of software that were available to them" (p. 28).

This issue is exemplified in a case study of a 'typical' experienced middle-school teacher (Myhre, 1998). Within her mathematics programme, the predominant form of computer use was of drill-and-practice packages to consolidate step-by-step procedures for solving standard types of problem. The teacher saw such software as permitting students to make repeated attempts to solve a problem correctly without having to seek her help. Additionally, however, her mathematics programme included some Logo sessions in which students experimented with the software as a means of drawing. The teacher saw these sessions as preparing the ground for future work in geometry. Both forms of computer activity were valued by the teacher as releasing students from what she characterised as 'boring paperwork', and providing an alternative style of working which they enjoyed. Although the predominant use of skill-focused software reflected the salience in the teacher's view of mathematics as "rules and processes that would lead the students to the right answer if they were applied correctly" (p. 98), the more exploratory use of open-ended software resonated with her aspirations to promote students' "understanding of mathematical concepts" (p. 100).

A second case study exemplifies the use of technology to support a 'constructivist' teaching approach of the type found elusive amongst mathematics teachers in the nation-wide survey. This study focused on the classroom practice of a high school mathematics teacher who employed graphic calculators and laboratory devices[1] in teaching pre-calculus classes (Doerr and Zangor, 2000). Described as "confiden[t] in her knowledge about the calculator's capabilities and its potential for student learning", the teacher made flexible use of the technology and encouraged her students to use it freely. The way in which the teacher structured and shaped classroom activity and calculator use reflected her particular concern that students should interpret and explain mathematical processes and results. In particular, the way in which technology was used to effect routine elements of measuring, calculating, tabulating and graphing, was aimed at focusing students' attention on overarching issues, and facilitating the testing of conjectures and checking of results. This teacher valued technology as a means of aiding – and persuading – students to find meaningful responses to problem situations.

Overview of prior research

In summary, some valuable studies of the integration of computer use into mainstream teaching practices and teacher thinking have already been conducted. Surveys have provided a simple profile of the forms of computer use taking place, and of pedagogical orientations associated with them. However, evidence from these surveys themselves indicates some of the limitations of employing preconceived models and questionnaire items to capture teachers' perspectives and practices. Case studies based on interview and observation have offered a useful complement to such surveys, providing more naturalistic and holistic accounts of the perspectives and practices of individual teachers. However, the focus on individuals restricts the capacity for analysis across cases, and thus for building theories and models with stronger potential for transferability and generalisability. There is a need, then, for naturalistic studies which provide analyses of the perspectives and practices of groups of teachers across different settings. Equally, all the studies discussed above were conducted in the United States. England is another educational system with the sustained experience of widespread use of computers in school mathematics which offers the scope for conducting such studies; one where the use of computer-based tools and resources, commonly referred to as information and communication technology (ICT)[2], has been a component of the national curriculum for over a decade.

DESIGN OF A STUDY OF TEACHER CONCEPTIONS OF ICT USE

Inquiry rationale

The study to be reported here aims to develop this line of enquiry into how teachers conceive their incorporation of use of computer tools and resources into mainstream mathematics teaching. It complements the studies discussed previously in a number of ways. First, in accordance with suggestions already made, it extends such research beyond the United States, and pursues an approach intermediate between large-scale surveys and individual case studies. More fundamentally, this study reflects a somewhat different theoretical orientation, and employs rather different methods, as will be explained. These features enable the study to enhance the research corpus by increasing the scope for triangulation of generic conjectures across studies conducted in different educational settings, making different assumptions and employing different methods. In summary, then, the aim of the study is to develop a model – perhaps particular to place and period – of what practitioners conceive as the successful use of computer tools and resources to support mathematics teaching and learning.

Theoretical orientation

Two intellectual traditions have influenced the theoretical orientation of this study, without providing a detailed apparatus. The first is a naturalistic tradition of research into teaching which has insisted on the necessity of attending to teacher thinking, not just as an important component of the phenomenon under investigation, but as a crucial source of constructs with which to build more potent theories. Rather than seeking to analyse teaching in the preconceived terms of prescriptive models or disciplinary theories, this approach seeks to elicit teacher accounts of their practices, and from these to articulate models which take account of how practitioners themselves interpret their experiences and formulate their actions (Brown and McIntyre, 1993; Cooper and McIntyre, 1996). This approach has led to a greater appreciation of the complexity and uncertainty of teaching. What emerges is not a deterministic world in which standard procedures can be applied with predictable effect, but one in which teachers must continually read evolving situations and tailor strategies accordingly to reshape them. Given that the tendency has been to promote the educational use of technology in overly idealised – and deterministic – terms, there is a pressing need for naturalistic studies more directly grounded in the actuality – and contingency – of teaching. Hence, unlike the surveys cited earlier, in which teaching was preconceived in terms of 'didactic' and 'constructivist' pedagogies, this study seeks to develop theoretical constructs from the ways in which teachers themselves talk about their pedagogy.

The second tradition is a cultural psychological one concerned with collective systems and intersubjective processes of thought. This tradition emphasises not only the social dimension of thought, but the distinctive forms of everyday thinking. Socio-cultural theories of this type have been developed around Moscovici's notion of 'social representations' (Moscovici, 1981), Bakhtin's concept of 'ventriloquation' (Wertsch, 1991), and Geertz's analysis of 'simple wisdom' (Page, 2001). What they share is a concern to explore the content and form of 'common sense' thinking, focusing on public norms over personal variants. The contemporary circumstances of teaching also accentuate this point. In England – as in the United States – the subject department acts as a basic social unit within secondary schools for the formal organisation of teaching and the informal association of teachers. In England, too, recent educational reforms have led to departments playing a more active part in mediating between national regulations and classroom teaching, through the development of departmental teaching policies and detailed schemes of work. Hence, unlike the case studies of single teachers examined earlier, the unit of observation for this study is the school department, and the concern is with ideas shared within – and across – departments.

Working context

The opportunity to pursue the investigation to be reported here arose within a university/schools partnership which is seeking to operationalise ideas such as 'the knowledge-creating school' (Hargreaves, 1999) and 'warranted practice' (Ruthven, 1999). Briefly, a 'knowledge-creating' school is one engaged in a managed process through which professional knowledge is created, validated and diffused. Likewise, 'warranting' a pedagogical practice involves developing an explicit rationale for the practice in action, analysing its processes in operation, assessing their impact on student development, and refining the practice accordingly.

Developing the use of information and communications technology (ICT) to support subject teaching and learning was identified as a priority across the participating schools. The aim of the opening – formative – phase of the resulting project – conducted over the first half of 2000 – was to identify and analyse what students and teachers saw as successful practice in this area, in anticipation of a second – developmental – phase in which promising approaches would be developed and investigated in greater depth. Because the research partnership aims at promoting cultural change within participating schools, and this particular project at supporting pedagogical development, it was important to adopt approaches conducive to these ultimate goals.

Thus, while this study aims to make a scholarly contribution to our understanding of teachers' pedagogical thinking, the wider project also has a very practical goal of stimulating and informing practitioner reflection on the use of technology to support teaching and learning as part of a

TABLE I

Basic profiles of the participating schools

School [Code]	Age range of students	Sex of students	Number of students	Proportion of students entitled to free school meals	Proportion of students achieving GCSE exam benchmark
Community College [CC]	11–16	Mixed	961	15%	54%
Girls School [GS]	11–18	Female	1050	13%	67%
Language College [LC]	11–18	Mixed	1316	7%	62%
Media College [MC]	13–18	Mixed	1500	5%	69%
Sports College [SC]	11–16	Mixed	1023	2%	74%
Technology College [TC]	11–16	Mixed	1237	5%	68%
Village College [VC]	11–18	Mixed	1305	5%	51%

programme of development and research aimed at assisting educational improvement. Thus, this study is intended to be a study of *teaching*, as much as of *teachers*. The resulting model is not viewed as definitive and conclusive, but rather as providing a starting point for further development. Such development will depend on creating productive interplay between the craft knowledge of teachers and the theorised knowledge of scholars, and between processes of knowledge creation within teaching and within researching, leading to the realisation of a correspondingly refined model (Ruthven, 2002).

Participating schools

This working context dictated an opportunity sample of participating schools. It is important, therefore, to clarify their broad character, and relate them to English secondary schools as a whole. All were located within 80 kilometres of Cambridge. Basic information about each is given in Table I. Pseudonyms have been adapted from official designations, and the corresponding abbreviated codes will be used – in due course – to indicate sources for quoted material. Three of the schools – Girls School, Language College and Village College – covered the full secondary age range from age 11 to 18; one school – Media College – was an upper school with students entering at the later age of 13; and three schools – Community College, Sports College and Technology College – covered only the compulsory stages of secondary education to age 16 (with students then transferring to other institutions to continue voluntary study to age 18). Although some of the schools had – or aspired to – specialist status – as indicated in their pseudonyms – none operated a selective admissions policy, and all but one – Girls School – catered for both male and female students.

Further relevant data have been extracted from official performance tables (dated 2000) and inspection reports on individual schools (dated between 1996 and 1998). The proportion of students entitled to free school meals is a standard indicator of social disadvantage: two schools – Community College and Girls School – lay close to the national median for schools (14%) on this index; and the remainder showed markedly lower levels of disadvantage. The proportion of students gaining the benchmark of 5 or more higher-level GCSE examination passes at age 16 is a standard indicator of academic success: two schools – Community College and Village College– stood a little above the national median for schools (45%) on this index; and the other five showed markedly higher levels of academic success. Against national norms, then, the schools in this opportunity sample were relatively socially advantaged and academically successful; ranging from Community College – around the national average in terms of social disadvantage, and somewhat above in academic success – to Sports College – highly favoured in both respects.

Investigative strategy

The first phase of the partnership research project employed group interviews with students – in different age groups – and teachers – in different subject departments – to identify and analyse what students and teachers saw as successful practice. In particular, this study draws on the group interviews held with the mathematics departments in the participating schools. Given other heavy demands on teachers' time, it was quite difficult to schedule these sessions, and in some schools not every member of the department was able to participate. Lasting between half an hour and an hour, the sessions were facilitated by members of the project team – either from the university or another participating school – who were not themselves teachers of mathematics. This 'outsider' audience was intended to encourage department members to articulate ideas which might otherwise have been taken for granted. Of course, this may also have encouraged some degree of simplification and introduced an element of presentation. Nevertheless, such effects would not be unhelpful to an exploratory study of this type, aiming to sketch out the main lines of accepted thinking about successful use of ICT.

The interviewer adopted a positive stance, with the main prompt requesting examples of ICT use which participants felt had been successful in supporting teaching and learning. Typically, this elicited accounts of several examples, often guided by attention to the content of the departmental schemes of work for each year group. Such accounts were not always wholly positive, sometimes tentative, and occasionally speculative, and this will be given further consideration in due course. When examples started to flag, a secondary prompt asked participants what impact they thought ICT was having on teaching and learning in mathematics. Later still, and time permitting, further prompts asked participants about the impact of government initiatives on their use of ICT, and their ideas about future developments in this area. This sometimes elicited further examples of, or comments on, current practice.

A number of features of interview talk indicated a relatively strong appeal to shared ideas and common experiences. Sometimes this was through explicit allusion, often related to aspects of departmental 'schemes of work'. It was signalled also by the way in which colleagues would affirm, reinforce and extend each others' comments (and something of this will be seen in later quotations where / indicates a switch of speaker). Finally, use within interview talk of the first person pronoun 'we' against 'I' (incidence of 1.6 per cent against 1.7 per cent) was particularly high when compared to the British National Corpus of talk in similar circumstances (incidence of 1.2 per cent against 2.2 per cent). Where potential breakdowns in consensus did occasionally emerge, they tended to be repaired by finding common ground, or simply passed over.

The audio-taped sessions were transcribed and edited into relatively short units of talk (guided by switching between speakers, and closing or conjoining within speeches) nested within more sustained episodes (guided by shifts or breaks in the focus of attention). This permitted analysis at the three levels of basic unit, sustained episode, and complete interview. The resulting transcripts were imported into a computer database to facilitate a recursive process of thematic organisation through constant comparison. This led to the construction of prototypical categories, grouping together related material. The goal was to identify well developed themes running across transcripts, and to examine relationships between these themes. This led to the omission from the final category system of some marginal ideas which did not meet these conditions, and could not be convincingly assimilated to other themes.

An important issue was how to treat examples where the success of ICT use was qualified or rejected in some way, or claimed in speculative terms, such as:

> I've taken a GCSE class in to use the Internet revision, but again, I couldn't say that was successful because the Internet is a bit slow, at the moment. . . I've used that with them after school, but it was slow after school as well. The success of it isn't quite what we'd want it to be, but it would be successful if it was faster. [VC]

While this particular instance of ICT was judged unsuccessful, there is a further suggestion that this type of use would be successful under other circumstances. However, it was considered important that any resulting model should be grounded in actual experience of success, not rationalisation of failure, or speculation beyond experience. For these reasons, only material meeting this condition was used, although the actual success might be qualified in some way.

CIRCUMSTANCES OF ICT USE IN THE PARTICIPATING DEPARTMENTS

Before proceeding to the substantive issue of the use of ICT to support teaching and learning, some consideration needs to be given to the circumstances of ICT use in the participating mathematics departments. Incidental comments during the interviews illuminated these circumstances, as did other conversations noted and material gathered during site visits. Issues of access to ICT facilities, planning of ICT use, student ICT capability, and types of software employed, will be examined in turn.

Access to ICT facilities

At the time of this investigation, important differences between departments were apparent simply in terms of access to ICT facilities, as summarised in Table II. In all of the schools, making use of computers to support teaching and learning depended primarily on gaining access to specially equipped ICT rooms. At Media College and Technology College there were departmental ICT rooms, with the latter particularly favourably resourced:

> Within the maths department, we have a dedicated suite just for maths, with sixty machines, so that we can have four classes with two to a machine and that's a bookable resource just for maths. There are other bookable rooms within the school, but we're generally self-sufficient. [TC]

TABLE II

Access to ICT facilities in participating mathematics departments

School [Code]	Department IT room	Scheduled access to school IT room	Opportunist access to school IT room	Department set of palmtop computers	Department set of graphic calculators
Community College [CC]			✓		
Girls School [GS]			✓		✓
Language College [LC]		✓		✓	✓
Media College [MC]	✓				✓
Sports College [SC]		✓			✓
Technology College [TC]	✓				
Village College [VC]			✓		✓

At Language College and Sports College, classes in the mathematics department had scheduled access to school IT rooms, although in the latter case this was only at the end of the academic year:

> The suites of computers are almost entirely taken up for... lessons with the IT department... We can only get into those currently when [final year classes] leave to take their exams, because that frees up some time, so we nip in there. [SC]

At both Community College and Girls School, there was a school ICT room adjacent to the mathematics rooms, and teachers relied on opportunistic access. In the latter case this could be problematic because the IT room also served as the normal teaching room for a colleague:

> To use [computers] we've got to swap class[rooms] and [named teacher] was having to swap for two terms which has an effect on the learning of her class, so that's an obstacle. [GS]

At Village College, such opportunistic access was possible only for some classes, because heavy demands on the school ICT rooms meant that they were often timetabled for use in teaching specialist ICT courses or vocational options.

In many of the schools, departments owned class sets of hand-held machines, making it relatively easy for teachers to give students access to such technology in the normal classroom setting; sometimes also with projection facilities. In particular, there was widespread use of graphic calculators – in all schools except Community College and Technology College – notably for graphing purposes. The department at Language College also had a class set of palmtop computers which permitted classroom use of spreadsheets.

Planned use of ICT

Departments were conscious that developing ICT use had involved learning on their part, sometimes under an element of duress. In particular, an important issue was developing clear curricular relevance for ICT use:

> We did have a spell when we had a period booked once a fortnight in the IT room, and you basically had to take a class in there. And what we did was put the relevant [courseware] into the module that we were doing at the time / We kind of forced ourselves to use ICT, by programming that in there. We got to discover things that we might not have used otherwise. [VC]
>
> It's been very much... just encouraging people to get on and use the machines... whereas now we're realising that it's far more powerful. It's not just a "go and use it because it's your turn on the timetable", you know like "I'll find something to do". It's more like "When's my opportunity because it's actually an integrated part". [MC]

Developing ICT use relevant to the curriculum, and doing so consistently across a department, entailed building it into the departmental schemes of work which guided teaching:

What we should do really is build these things into the schemes of work a bit more. At the moment, it's a little bit "What shall I do today?" sort of thing. [GS]

We're currently involved in a re-write of schemes of work and it's going to be much more heavily ICT referenced. [CC]

We rewrote schemes of work a couple of years ago... We took the opportunity to integrate into each module some IT activities written by members of staff. [MC]

However, where access to facilities was problematic, integration of IT use into the scheme of work was not viable:

It's up to individuals... Really it's difficult to force people to use it when we haven't got the facilities to do that with. [VC]

Equally, where access was limited, incorporation of ICT use could only be on a modest scale:

At the moment we're in the situation where we've had a week's worth of Logo work with Year 8, we've had a week's worth of spreadsheet work with Year 9,... and then graphical calculators are there for lots of different uses, but that's the lot. [SC]

By contrast, where there were few constraints on access, the scheme of work could specify both an extensive core of ICT use, and further optional possibilities:

Within the scheme of work... there are specific topics to be done on the computer, and we all follow the same scheme of work, we'll use the computers for the same things, so all pupils will have the same access. / But there is a distinction between those items which are considered to be part of the programme of study, which everybody will use, and other applications which are possible ways. We have various software on specific topics or whatever which are written in to our activity packs, which one teacher may choose to use, or may choose not to use. So there is a basic entitlement for all, and on top of that individuals can make use of other applications. [TC]

However, seeking to incorporate extensive use of ICT revived the basic tension:

I always feel it's my job... to push as much IT into the scheme of work as is possible. But that's always battling against... am I actually achieving anything by putting the piece of IT into that particular module of work, or is it really just being there for the sake of being there? [TC]

An emphasis on ensuring that ICT use matched the mathematical objectives of the scheme of work was general:

Their IT stuff that is arranged at the beginning of the year, it is arranged to hit the modules of work at the time that you are doing that topic. It's not just that you must go down to the IT room, just go and use this. It is part of the scheme of work. [LC]

The policy is that we shouldn't use it for the sake of it. Really, I suppose that's the main all-encompassing thing. [VC]

It's important that it fits in with what they are doing, rather than just giving it. [CC]

It has to support what we're doing. [SC]

Student ICT capability

However, the demands of equipping students to make use of ICT could be in tension with the mathematical productivity of lessons; although the growth of quite separate ICT courses for all students, accompanied by wider use of ICT across the curriculum, was alleviating this:

At times it feels like we're doing the job of an IT department and a maths department... and I think trying to get the balance right between "Okay, here's a maths problem. Use what you know to solve it" or "Here's how you use IT to do something"... / It's getting better, though... [Now] they're having timetabled, actual IT lessons and that's helping. They'll come through with those skills and we'll be able to use [them] in maths. [MC]

We have moved from a position when... we used to teach spreadsheets... [And] by the third lesson you might be ready to actually do something with the spreadsheet which was fairly mathematical. Whereas we're in a position now where when we do spreadsheet activities... we're able to actually go into the classroom and say "Right, this is the activity we want to do. Load up Excel" and the pupils will know what I'm talking about. [TC]

Other comments brought out the way in which such changes were reducing the drag of technical difficulties on classroom work:

So they are coming to us now, more confident and more educated. So you can just say "Start the machine" and they do just do it. You don't have to say, you are not walking round saying "No, you have got to come back and click on that". So you are very quickly into lessons... /... It's just "Get started. Go." And they are much, much more comfortable and much busier. [LC]

I think it's getting better because now... they've all done Excel in IT as well as in Maths... It used to be an awful problem. They were always getting stuck, something quite trivial in Excel, getting lost in the spreadsheet. Well now that hardly ever happens. [GS]

In this respect, the situation regarding graphic calculators was rather different, as their use was generally restricted to mathematics:

They also need to be brought up on the calculator really. To suddenly buy one and not be used to really going into it is very difficult. They really need to get it, say Year 11, and be used to it before we start the course in Year 12... / They really ought to be entering the sixth form, being familiar with at least a model of graphical calculator. So that is why it is important for us to give more access to it in lower school. [LC]

An important related factor was the power and ease of use of ICT facilities, which could be lacking if equipment was sufficiently old to differ in operation and capability from that used by students at home:

While they've helped the learning to take place, I guess, mathematically it's not been so inspiring to students who've had to work on old, outdated computers. We've also had to spend a lot of time and energy in training the students to use these computers so they can get out of them what we want. This has meant they're quite reluctant to do it because they've got PCs at home and they know they do it a lot quicker and a lot easier that way. [MC]

I think IT was dead on its feet in this place, until this year... It was dreadful. It was appalling... This year we have gone in. The technology is there... The kids are captivated by it. They love it. They know about using machines. I can now spend my time helping kids understand what they are supposed to be learning, rather than "Oh God. What button did you press? How do we get back to where you were?" Reset the machine. [LC]

Those departments making less extensive use of ICT expressed more concern about technical matters. In one case, a shift towards the use of commercial courseware was seen as a solution:

It was all quite complicated spreadsheet work where you started from scratch. We didn't have the chalk face programs [courseware] which simplify it all... It was all quite difficult stuff, apart from Logo. [CC]

In another department, even courseware was seen as unpredictable and uncontrollable:

The published software. You glimpse at it when you get samples and it seems alright, but when you get the children, you get thirty children using it. / It does work like that! / They always do things that you don't expect and the unexpected happens. It either crashes or brings up things that are way beyond their capabilities... Whereas if you had set the thing up yourself it may have been simpler, not as flashy, but it would be doing the job that you specifically wanted it to do. If it was finding out the term of a sequence and you'd set up a spreadsheet that would generate sequences and they would practice and practice, you'd have control over the input, how difficult the sequence was, and things like that. [VC]

This approach of authoring tightly framed courseware had actually been carried through in a third department:

That involved interactive spreadsheets... [which] worked much like a collection of linked web pages, and so on each page there'd be a teaching point, something that they would need to know about, or need to be able to do, and there'd be perhaps some text to show them how to do this, and then there were some questions to demonstrate that they could do it and they could try it out. When they had completed a question, it marked it for them and told them whether they were correct or incorrect. If they were incorrect it occasionally gave feedback as to

TABLE III

Types of software used in participating mathematics departments

School [Code]	Courseware	Graphware	Logo	Spreadsheet	Other
Community College [CC]	✓		✓	✓	
Girls School [GS]	✓	✓	✓	✓	
Language College [LC]	✓	✓	✓	✓	Database
Media College [MC]	✓	✓	✓	✓	Word-processor
Sports College [SC]	✓	✓	✓	✓	BASIC
Technology College [TC]		✓	✓	✓	D/base, W/proc
Village College [VC]	✓	✓		✓	

where they may have gone wrong. For example leaving an equals sign off the beginning of a formula. And then when they had completed a sheet successfully, a hyperlink appeared for them to move on to another, or to the next topic or next learning point. And then interspersed with these, when they'd acquired enough knowledge and enough skills, there was investigative work that would then use the things that they'd learnt up to that point. [SC]

Types of software employed

The national curriculum, introduced to English schools from 1989 onwards, was a central influence on departmental schemes of work. The original curriculum specified four types of computer use in secondary mathematics (Department for Education and Science [DES], 1989):

- Use spreadsheets or other computer facilities to explore number patterns;
- Generate various types of graph on a computer or calculator and interpret them;
- Use computers to generate and transform two-dimensional shapes (Exemplified as: Use Logo to draw regular polygons and other shapes);
- Insert, interrogate and interpret information in a computer database.

Although the curriculum itself had been revised subsequently, these specifications had not changed substantially, and they had set the agenda for implementation of the new curriculum. Consequently, as shown in Table III, the types of software used in departments were fairly uniform. The term 'courseware' refers to software designed to 'teach' or 'test' a particular mathematical topic; and 'graphware' to graphing tools whether on computer or calculator.

TABLE IV

Inspection comments on use of ICT in mathematics departments

School [Code]	Relevant comment(s) from school inspection report(s) [Year]
Community College [CC]	Information technology skills are under-developed. [96] The recently introduced use of computers to support learning does much to improve the quality of learning. [99]
Girls School [GS]	One of the main information technology rooms is adjacent to the department and is used regularly by the mathematics department. [97]
Language College [LC]	[Pupils] use both Logo and spreadsheets within mathematics. [96]
Media College [MC]	The pupils benefit considerably through their confidence with computers and the opportunities they have to use them to enliven their mathematics lessons. [96]
Sports College [SC]	Access to computers is very limited, though the work undertaken is good. [98]
Technology College [TC]	Very effective use is made of information technology to support pupils' learning. [97]
Village College [VC]	Some use is made of computers to support learning, but this is under-developed at the moment. [97]

Summary overview of departments

Independent evidence drawn from school inspections suggests that differences in the use of ICT between the participating mathematics departments were relatively longstanding, as shown in Table IV. The brief comments of the inspectors are broadly compatible with our judgement that the departments were diverse, ranging from Community College, least developed in this respect, and Village College, where poor access to ICT facilities was a major constraint; through Girls School, Language College, and Sports College, where use of a range of computer-based tools and resources had been developed, but opportunities to employ these remained restricted by limited access; to Media College and, most prominently, Technology College, where there was regular, planned use of a range of tools and resources.

THEMATIC ANALYSIS OF REPRESENTATIONS OF SUCCESSFUL ICT USE

Having sketched the circumstances of ICT use in the participating mathematics departments, it is now appropriate to turn to the substantive issue: analysis of the teacher accounts of successful use of ICT to support the teaching and learning of mathematics. This section will introduce the thematic framework which emerged from this analysis.

Success and operational themes

The initial analysis focused on the *success themes* which were explicit or implicit in teacher accounts. It appeared that these themes could be conceptualised in terms of three related priorities, concerned with securing and enhancing the *participation* of students in classroom work, the *pace and productivity* of such work, and the *progression* in learning arising from it. Two quotations will be used to illustrate these success themes. The choice of these particular quotations reflects an emphasis on the part of the teachers. Although they referred to a range of ICT tools and resources, it was the use of various forms of calculator or computer graphing which attracted a disproportionate amount of comment. Equally, these particular quotations illustrate how these references to use of graphing technology covered the full age and ability range at secondary level: from the academically lowest class (the 'bottom set', containing those pupils judged weakest in the subject) in the youngest age cohort ('Year 7', aged 11/12); to the academically highest course ('Further Maths') in the oldest age cohort (Years 12 and 13, aged 16/18):

> We've used spreadsheets in Year 7 and 8, to enable them to look at handling data, because they can quickly get tables and produce charts that are much better quality than those that they can produce themselves. I've got the bottom set in Year 7 and it can take them the whole lesson to draw a bar chart. So it's particularly successful from that point of view... because they don't have to draw all the axes so much, and it doesn't take them so long to develop the ideas because they're not having to spend a whole lesson drawing something. They can draw twenty graphs in a lesson and actually see connections, rather than spend twenty minutes drawing the axes and then twenty minutes talking and then twenty minutes drawing all the graph. [VC]
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> It saves a lot of time as well with the Further Maths and the graphing that we did. It would have taken forever to actually plot all the points and see what happens when you transform certain shapes. Whereas it was done in a flash and they could see and they learnt an awful lot. So then they were ready and they'd accepted it because they'd seen it happening... Whereas it would have taken many lessons if we'd actually plotted all these graphs, they'd have just got bored by it. So that definitely helped, just kept the pace going. [MC]

Both these quotations appeal to comparisons with similar activities conducted without access to computer graphing tools. Issues of *participation* are framed in terms of overcoming disengagement ("twenty minutes talking") and demotivation ("just g[etting] bored by it") on the part of students. Issues of *pace and productivity* are framed in terms of time saved and pace maintained ("it doesn't take them so long"; "it saves a lot of time", "it would have taken many lessons", "just kept the pace going") and work produced ("they can draw twenty graphs in a lesson"). Issues of *progression* are framed in terms of ideas being formed ("actually see connections"; "see what happens when you transform certain shapes") and embraced ("they'd accepted it because they'd seen it happening"), and in more general terms of development and learning ("to develop the ideas"; "they learnt an awful lot").

These success themes provided a useful basis for further analysis to identify the technological affordances and mediating processes which teachers saw as underpinning these forms of success. From this further analysis, ten *operational themes* emerged as prominent across the departmental interviews. In discussing each of these operational themes, relevant extracts from transcripts will be used to provide brief illustrations. Often these extracts also invoke themes other than the one under discussion, but it would be unduly cumbersome to catalogue these on every occasion. However, as the flow of argument permits, reference will be made to associated themes.

Ambience enhanced

The theme of *Ambience enhanced* associates ICT use with change, difference or variety in working ambience. At one level this was often a matter of change of working location -from ordinary classroom to computer classroom- and correspondingly of work organisation:

It's variety and the kids love to go to the ICT room to do some work. . . They like a change from the routine of the classroom situation. [CC]

It's just a different way, rather than being sat in the classroom. [MC]

Where technology use was seen as something of a break from 'routine', such motivational effects could be attributed to the novelty of the situation, or to the conferment of privilege:

The novelty value motivates them. [CC]

It's not having to write everything down, and they see it as a privilege to go in and use the computers, so already they're happy, they put more effort in just because it's a change. [MC]

They seem to respond well to that in the sense that they couldn't believe that people in that group were being allowed to use those things, and they were quite taken with the idea of producing graphs on a screen. [LC]

Typically, work in the computer classroom involved students interacting directly with a machine; but also, given the normal situation in which the number of class members was around double the number of machines available, students often worked not individually but in pairs:

I think there is a different style of work that's going on in some instances. I mean, necessarily there is collaborative work in which the pupils are working in pairs [TC]

But even in an ordinary classroom, with teacher-led activities directed at the whole class, an interactive style of ICT use created a distinctive feel:

I've only this week been using a program, just a short one, for the students to input a number, which it then outputs following a rule, so they can try and spot the rule by choosing different inputs and seeing what the outputs are. They quite enjoy seeing things done in a different way. [VC]

It seems that it was such exploratory, interactive styles which led to ICT use sometimes being characterised –in a wholly unpejorative way – as 'playing around' and the devices themselves as 'toys':

They liked the idea of playing around on the calculator. [LC]

They enjoy it as a motivating toy. Especially graphic calculators. They think they're like toys, don't they. [GS]

Teachers talked of the 'variety', of the 'difference', even of the 'other dimensions', associated with ICT use, and related these to securing and enhancing student participation in classroom activities:

Because you're teaching in a variety of ways and there's another dimension there, I think it just improves the motivation. [TC]

And they like using it as well. I think it's added extra dimensions. [GS]

Restraints alleviated

The theme of *Restraints alleviated* associates ICT use with the alleviation or mitigation of factors restraining or inhibiting the participation of students in classroom work. One important sub-theme (related to the 'play' sub-theme of *Ambience enhanced*) concerned the capacity of ICT use to prevent 'work' becoming 'drudgery':

The lower ability students are far happier playing on a computer and getting a computer to draw the graphs or solve different things rather than having to do it all by hand. [MC]

The key thing about the calculators or any ICT applications being able to take away the drudgery out of doing the calculations. [SC]

As an earlier comment about "not having to write everything down" hinted, a further sub-theme focused on the way in which ICT use is often associated with a reduction – or removal – of the writing demands – physical and intellectual – of much conventional classwork; demands which may deter or challenge some students:

I think the main advantages are that it doesn't involve them doing lots of writing, which is often something that the lower attainers in maths are unhappy doing. [SC]

Even really weak ones... actually do quite well on computers, don't they, because their literacy isn't challenged too much. [GS]

Another specific area where weakness in student capacity was seen as alleviated by access to technology was the drawing of graphs, particularly with less academically successful students in 'bottom sets':

Going back to straight line graphs, they are particularly weak at drawing just the axes and just the graphs, so it actually gets them to see what it should look like, and they can get printouts and copies in their books and have some nice neat work to look at. [VC]

You go in with bottom sets and you tried before that to get them to draw graphs, but now you can go in and just draw the graphs on there, and they can see the gradient. It is so much better. It is just impossible otherwise with bottom sets... It is just very neat for any set, but bottom sets in particular who can't draw graphs very well. [LC]

This last quotation continued by pointing to improved opportunities for participation by students with physical disabilities:

It is particularly useful as well for some of our disabled students who cannot physically draw a graph but they can work a keyboard. [LC]

A final sub-theme concerns the way in which ICT use changes the status of mistakes, not only by facilitating their correction, but by removing evidence of them which might attract unwelcome – and demotivating – attention from the teacher:

If you're working in an exercise book, then I think some students don't respond well to situations where they make mistakes. Before you know where you are there's a scribble all over their books. But if you're working on a screen, then it's just sort of click and then you're off again. [CC]

They don't seem to mind getting things wrong either. Whereas, if they got it wrong in their book, particularly children who are already feeling a little bit, lacking confidence let's say, mathematically. To get something wrong in their book often results in "Oh, I'm not doing maths." But if they get it wrong on the screen, okay and they just go back to the drawing board and do it again. And they will do it over and over again. / If it is deleted on the screen and it is not there anymore, it is forgotten. / And you don't write all over it. [LC]

Underlying this change is what might be termed the 'provisionality' of results in many technological environments: their openness to revision, and their transience of record. The central focus of the next theme is on the strategic affordances of provisionality.

Tinkering assisted

The theme of *Tinkering assisted* focuses on how the provisionality of many ICT results assists forms of tinkering to improve them. The most basic idea was that of ICT supporting self correction by students, prompted either by the machine producing a response different from that expected, or by an evaluative mechanism built into the system:

And they could see for themselves what was wrong with it, they could correct it. [CC]

And they can see straight away when it's wrong. / Exactly and they can see. / And they can correct themselves. [GS]

Whereas such 'corrections' were applied to what were intended as definitive solutions to a given problem, the provisionality of ICT facilitates 'trials' of more tentative solutions, and hence a corresponding shift in strategic logic. The idea of 'trial and improvement' strategies formed part of the national curriculum being followed by these schools, and teachers made reference to the way in which use of ICT supported such strategies. More generally, they pointed to the way in which use of ICT afforded a more experimental approach to tasks (related also to the invisible mistakes sub-theme of *Restraints alleviated)*:

They are more prepared to have a stab at something and get it wrong because not everyone can see it's wrong and they'll keep trying until they can get it right. [VC]

They can do more investigative work, can't they. If something doesn't work, then they can try something else. [GS]

Comments also suggested that this readiness to experiment was characteristic of many students, encouraged by other school experiences of using ICT:

The IT programme... encourages them to experiment and do their own thing. [GS]

Watching young people, often with software we're not absolutely happy with... they will find out things we haven't discovered because they're not afraid of pressing a button and seeing what happens. And I think they learn sometimes much faster through that than the structure that we're trying to give them. [VC]

Arguably, it is awareness of this tinkering style of ICT usage which underpins earlier comments about students "playing on a computer" or "playing around on the calculator", under the *Ambience enhanced* theme.

Motivation improved

The theme of *Motivation improved* associates ICT use with the motivation of students towards classroom work. This idea has been present in many of the quotations which have already been used to illustrate earlier themes. Teachers have commented on what students 'love', 'like' and 'enjoy' in relation to using ICT; likewise on what 'motivates them', on what they 'respond well to', and on what they are 'quite taken by'. These sub-themes of student appreciation and attraction will not be illustrated further here. However, one further sub-theme does merit attention. It suggests that by assisting or permitting students to display – and to be seen to display – greater capability, use of ICT can help build their self-confidence. Across more extended sections of transcript, this idea was associated with the earlier themes of *Restraints alleviated* and *Tinkering assisted:*

> I think they can then start to feel more positive about themselves and their work, because often they can just get worse and worse with their untidiness because they know they can't do it so they can't be bothered to try. If they see what ICT can help them do then they might be encouraged to try it a little bit more themselves and then try to improve their own work to that standard. [VC]
>
> And some of them, although their mathematical ability is quite poor, they are actually quite good with using computers and they get like a bit of a buzz from that. They can sort out problems and they are not struggling at the bottom any longer. [LC]

Engagement intensified

The theme of *Engagement intensified* associates ICT use with deeper and stronger student engagement in classroom work. Clearly this theme is closely related to *Motivation improved*. Likewise, illustrative quotations in previous sub-sections have noted students being "more prepared to have a stab", to "put in more effort", to "keep trying". The following quotations draw together many of the ideas associated with this theme – good behaviour, attention to classwork, and a degree of independence and persistence in it – and the second illustrates the linkage with *Motivation improved:*

> They are working more, they're not wasting so much time, they actually do get on with it, the majority of children will get on with it, rather than do one question, turn around and I've got to tell them what to do so they are learning something. [CC]
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> I think it is almost that they have got a fairly short focus and attention span, and they like the immediacy of what is happening, and they are always okay, they are always better behaved, or I think they will be when they arrive, quite focused. [VC]

A further quotation illustrates interplay between the themes most closely associated with participation. It relates aspects of *Restraints alleviated* ('writing difficulties' and 'pen to paper motivational problems') to *Motivation improved* ('really enjoy doing', 'still very happy, still very, very keen') and *Engagement intensified* ('happy just to set it up themselves and to sit there themselves and tap away'):

The pupils really enjoy doing [numeracy package] for a number of reasons. Lots of them have writing difficulties. Lots of them have pen to paper motivational problems and this gets around all of those and means that they can get straight at it; "Okay I've got to multiply that by 100"; and they can tap, tap, tap and it's there. And they can get feedback immediately on how they're doing. . . the ones who are nervous know. . ./ They'll spend five minutes on the computer completing a small amount of numeracy practice and they're quite happy just to set it up themselves and to sit there themselves and tap away, and "I've finished" and then that'll be checked and they'll go back to [other work]. With this particular year group, I started with them in Year 7 using it, and Year 9 they're still very happy, still very, very keen. [SC]

This extract sets the scene for forthcoming themes, where the intensified engagement of students emerges as one factor contributing to improved pace and productivity of classroom activity.

Routine facilitated

The theme of *Routine facilitated* associates ICT use with facilitation of relatively routine components of classroom activity, allowing them to be carried out more quickly and reliably, with greater ease, and to higher quality. In this respect, aspects of *Routine facilitated* underpin some of the phenomena already noted under the theme of *Restraints alleviated*, and there was a clear intertwining of these themes in some sections of transcript. What characterises *Restraints alleviated,* however, is a focus on particular factors inhibiting the participation of persons, whereas *Routine facilitated* focuses on factors supporting the execution of tasks. The sub-themes of speed and ease were the most prominent, and these are illustrated in the quotations below:

Using the calculator. . . can speed things up when you are doing standard deviation and stuff like that, you can just wham in the numbers and they can get their estimates, can't they, their unbiased estimates and tests and things. [LC]

With a [graphic calculator] it is much easier because you have got zoom boxes, and you can work out the gradient of a particular point just by clicking on it very very easily. [LC]

Speed, accuracy and the ability to superimpose several things on the same set of axes. And you know that you're getting the right scale. [VC]

Such use of ICT was seen as particularly important in supporting the more extended project tasks which older students tackled as assessed coursework for the GCSE examination:

They're now used to using Excel for problem-solving, so that when they do their GCSE coursework they're used to using it, they know the shortcuts and how quickly a spreadsheet can sort things out. [MC]

It makes a big difference to coursework. . . Certainly with the older kids. . . they have the skills in Excel and Word, and when it comes to coursework we tend always to book the room and they go onto the machines and use them when they need to use them. [TC]

Two examples illustrate how use of ICT tools facilitated routine components of such tasks, highlighting the removal of important constraints on the strategies which students could pursue:

I had some Year 11 pupils doing GCSE Statistics who wanted to count lengths of words, and some pupils went through writing down, "Okay, that one's got 5 letters, that one's got 7 letters", from newspaper articles. I just kicked some ideas round with some other pupils who got a body of text from the newspapers' websites, transferred that into Excel, and then got the computers to count those, which is a fabulous use of ICT, and meant that once they'd got this set up, they can then analyse as much text as they wanted. So rather than someone poring over a page of the Guardian and counting numbers, they were able to do this in no time. [SC]

I've just worked this year using draw commands in Word for a bit of coursework called 'Square Moves' where they have to move counters around, and part of the coursework is to show their strategy for moving the counters, and I did it with a low ability group, and we got far better results I feel, because I showed them how to draw circles, and get the sizes right, fill them in different colours, and then once they'd set up the basic grid they could copy and paste the whole grid however many times they wanted, and then they just moved the individual counters around, and as a result of that they did far more work than with the old style of always drawing them out. [TC]

These quotations also illustrate the linkage of *Routine facilitated* with the next theme of *Activity effected.*

Activity effected

The theme of *Activity effected* associates ICT use with securing and enhancing the pace and productivity of classroom activity as a whole. The quotations at the close of the last sub-section illustrate the more global concern of *Activity effected* with pace ('rather than someone poring over a page... they were able to do this in no time') and productivity ('far better results', 'far more work'). The following quotation again illustrates the linkage of *Routine facilitated* to the pace aspect of *Activity effected:*

We've got a graph plotting package which isn't particularly sophisticated... but it makes it nice and easy for the kids to use... Actually drawing a graph and seeing it on the screen, they can very quickly see what's happened to the graph. So using IT in that respect, it makes a significant difference in the depth of understanding and the speed in which it takes to learn skills... You may get there in two lessons rather than three, so you can gain a lesson. [TC]

Here, too, there are intimations of the forthcoming themes of *Features accentuated* ("they can very quickly see what's happening to the graph") and of *Ideas established* ("it makes a significant difference in the depth of understanding").

Another quotation illustrates similar associations in which aspects of *Routine facilitated* ("the computer can do it really quickly for you") and *Restraints alleviated* ("it takes out all the laborious stuff"), and of the forthcoming theme of *Features accentuated* ("it's visual"), are related to facets of *Motivation improved* ("to make it more exciting") and *Activity effected* ("it's using it as a tool... to save time"), and to the forthcoming theme of *Attention raised* ("but you're still having to do the mathematical stuff yourself"):

It takes out all the laborious stuff... so if you're doing an investigation which is about... working out patterns, the fact that the computer can do it really quickly for you, and it's visual,... it's using it as a tool, like to save time, to make it more exciting, but you're still having to do the mathematical stuff yourself. [MC]

Features accentuated

The theme of *Features accentuated* associates ICT use with the provision of vivid images and striking effects through which features of mathematical constructs – or relations between them – are accentuated. The central idea is well summarised in the following quotation:

The actual immediacy of it and also the fact that it's living, it moves. [VC]

This 'immediacy' of response, already referred to in previous quotations, was one of the important sub-themes to emerge. Likewise the power of 'visual' representations:

It can be used quite effectively to offer clearer explanations, or visual ones at least. [MC]

A further sub-theme pointed to the dynamic of actions and their effects:

So you quickly see what is happening to the gradient and what is happening to the intercept. [LC]

Further quotations provide fuller illustration of these sub-themes of immediate response, visual presentation and dynamic effect:

Being able to put an expression into the calculator, saving it to a memory and then putting a number into a different variable, which then when you press the program, out comes the variable changed by the function, you start really giving these kids an understanding of what a variable is and how multiple factors can actually change things. [SC]

Whenever I do regular polygons... I say "Do you remember we did it on Logo", and... quite a few of them do remember it because they can picture the turtle actually drawing it and turning so many times. [GS]

[There are ideas in statistics] like the central limit theorem, which you can't prove to them... But you can load up this nice little program that will show you picking samples from any distribution and calculating the sample mean and drawing up a graph of them, and you leave it running in your classroom and they see it in action, they see the samples being picked, and they see it tending towards a normal distribution. So it's all those sort of concepts that are too difficult to prove, because they are beyond the range of the syllabus. You can see them visually. [LC]

These references to 'giving [students] an understanding' and to students 'remember[ing]' and 'see[ing]' also point to the way in which the phenomena highlighted in *Features accentuated* contribute to the progression in learning which is the focus of the forthcoming theme of *Ideas established*.

Attention raised

The theme of *Attention raised* associates ICT use with reducing or removing the need for attention to subsidiary tasks, and with avoiding or overcoming related obstacles, so as to better focus students' attention on overarching ideas and processes. The following quotations bring out the way in which processes already alluded to under the head of *Restraints alleviated* and *Routine effected* clear and smooth the way so as to raise and focus students' attention:

The key thing about the calculators or any ICT applications being able to take away the drudgery out of doing the calculations, so that you can start to access a higher learning point without the problems of making mistakes along the way, clouding the issue. [SC]

They don't have to spend ages drawing out the graphs. They can look at all the properties of different curves. /... They get over the stumbling block of actually drawing them in the first place. They can actually see what they are and concentrate on that aspect. [GS]

It's just a case of typing in equations and they can see looking at gradients and intercepts, it makes the learning of that so much more efficient and so much more successful because they can see what's actually happening rather than spending time with pen and paper drawing graphs. [TC]

A more complex extract, linking a greater number of themes, illustrates how elements of *Routine facilitated* and *Features accentuated* contribute to *Attention raised:*

One of the powers of ICT... is iteration because, again, it is the immediacy thing. With an ordinary scientific calculator, if you're going to do iteration you have got to put the complete formula in as a calculation with each new value, but with a graphical calculator you just press execute, which is tremendous... That again is all about focusing the mind on what is the important thing that we are trying to learn, and the thing about iteration is that your process homes in on a root, and if you go and spend a quarter of an hour putting in a calculation and getting it wrong on the calculator you are not going to see that it homes in on a root. That is the essence of iteration. It homes in. So a graphical calculator, it is complicated enough getting it in I know, but once you have got the formula in, it is just execute over and over, and you have got it, and you can see it coming up on the screen, you will see the digits clocking off. [LC]

Ideas established

The theme of *Ideas established* associates ICT use with the formation and consolidation of ideas. In earlier quotations the sub-themes of ideas being 'seen', 'understood', 'accepted' and 'remembered' by students have already arisen. The following quotation shows how aspects of *Features accentuated* support facets of *Ideas established:*

But also they were able to, I don't know, just see more clearly really about how things work. / Zooming in, that's right. / Trying to show them the shorter the chord, the closer it is to the curve. I find that really difficult to believe, without seeing it with my eyes, so I can understand the kids having a problem with that. And when you've got y equals x squared, a nice round curve and when you zoom in you get straighter and straighter and straighter, it is very convincing. [LC]

A final quotation illustrates the association of both *Features accentuated* and *Attention raised* with *Ideas established:*

When we're using Coypu [graphing package] for shifting curves, and that shows them very easily what altering the equation does to the curve, and that does help them tremendously. / [In] some cases with that it's actually kids who would have drawn a blank on that whole topic actually have a bit of insight. / It's a very, very difficult topic and so you can just actually get them to draw various curves, and show them what happens when you start altering the equation... Just the immediacy of it, actually means that it hangs together better, because if you see these results and then spend another fifteen minutes drawing a graph, the whole thing just feels nothing much, so it's just not better efficiency, but also it is actually sounder for the brain really, if it can see things more immediately. [TC]

Incidence and co-incidence of operational themes

These ten operational themes serve to summarise teachers' ideas about the affordances of technology in promoting their classroom goals. In effect, then, these operational themes point to components of a collective representation of appropriate and effective technology-supported pedagogy. So far, attention has also been given to associations between these themes where particular relationships have been made explicit in interview discourse. However, themes may be associated in collective representations without such relationships being formulated and articulated.

Another way of examining such associations – both implicit and explicit – is through analysis of the relative incidence and co-incidence of themes. The key indicators are assembled in Table V. Each row of the table reports on a particular theme. It first shows the number of transcript sections containing text units coded to that theme, and then the proportion

TABLE V

Incidence and co-incidence of operational themes

Operational theme	Number of sections coded to theme	Proportion of these sections coded to other operational themes									
		Ambience enhanced	Restraints alleviated	Tinkering assisted	Motivation improved	Engagement intensified	Routine facilitated	Activity effected	Attention raised	Features accentuated	Ideas established
Ambience enhanced	15	•	13%	0%	87%	20%	20%	7%	13%	13%	7%
Restraints alleviated	15	13%	•	27%	53%	40%	60%	20%	40%	33%	27%
Tinkering assisted	10	0%	40%	•	40%	20%	0%	0%	0%	10%	30%
Motivation improved	25	52%	32%	16%	•	28%	28%	8%	12%	28%	20%
Engagement intensified	10	30%	60%	20%	70%	•	30%	10%	10%	20%	20%
Routine facilitated	26	12%	35%	0%	27%	12%	•	35%	46%	31%	27%
Activity effected	11	9%	18%	0%	18%	9%	82%	•	55%	27%	45%
Attention raised	12	17%	50%	0%	25%	8%	100%	50%	•	50%	50%
Features accentuated	14	14%	36%	7%	50%	14%	57%	21%	43%	•	57%
Ideas established	14	7%	29%	21%	36%	14%	50%	36%	43%	57%	•

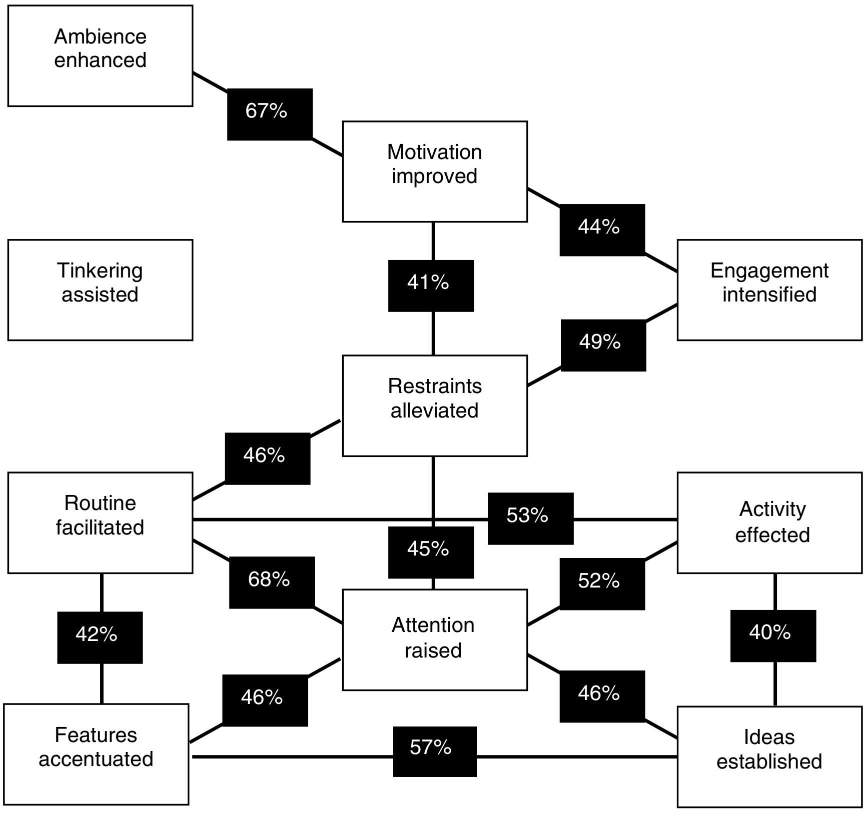

Figure 1. Major trends in co-incidence of operational themes.

of these sections containing text units coded to each other theme. The first column of data in the table, then, indicates the relative frequency of themes. It shows that Routine facilitated and Motivation improved were more prominent than others. The remaining data indicates the frequency with which themes were invoked alongside one another.

A more striking representation of the main trends is provided by the co-incidence diagram shown in Figure 1. This summary was derived by employing a simple index of the association between each pair of themes (a symmetric co-incidence coefficient, formed by computing the geometric mean of the relevant pair of proportions) to identify salient relations (operationalised as a co-incidence coefficient of 40% or more, corresponding to the upper quartile of such associations)[3]. The themes have also been organised spatially so that those processes afforded most directly by use of technology lie on the left of the diagram; those processes corresponding most closely to ultimate teaching aspirations – of participation, pace and productivity, and progression – lie on the right of the diagram; with potentially crucial intervening processes falling between. Broadly, the diagram divides into an upper component concerned with participation, and a lower component concerned with pace, productivity and progression.

In the upper part of the diagram, *Engagement intensified* was associated directly with *Motivation improved* and *Restraints alleviated* – and these with each other; and then indirectly – through these intermediate themes – with *Ambience enhanced* and *Routine facilitated*. Although *Tinkering assisted* was not associated sufficiently strongly with other themes to be linked to any of them, inspection of the corresponding row in the table indicates that its closest links were with *Motivation improved* and *Restraints alleviated,* and it has been positioned accordingly. In the lower part of the diagram, there is more extensive connectivity. The only direct associations missing are between *Activity effected* and *Features accentuated*, and *Ideas established* and *Routine facilitated*; but these are indirectly linked through *Attention raised*.

This diagram should be read cautiously. The themes constitute clusters of ideas which teachers invoked in discussing successful use of ICT. As such, they provide ideas likely to be valuable in guiding preactive and interactive thinking about teaching. However, the linkages simply indicate where there was substantial co-incidence of themes, not that they are inexorably associated. In particular, the diagram should not be read deterministically as implying that exploitation of the technological affordances on the left leads inevitably to achievement of the teaching aspirations on the right. As the transitional aspects in the centre of the diagram explicitly indicate, the teachers identified important intervening processes. Equally, all of these processes depend not on the mere presence of technology, but on its affordances being successfully exploited in ways appropriate to the situation. In effect, each construct represents a potentially desirable state to be aimed for. Hence, these themes are best taken as providing a system of ideas which might profitably be employed to inform the conceptualisation of classroom processes, and to identify important issues to be attended to in planning and steering such processes.

Pedagogical themes

On occasion, teachers discussed pedagogy in more holistic terms, using a corresponding language. This gave rise to two further *pedagogical themes* concerned with – on the one hand – 'investigative' activity and – on the other – with activity aimed at providing 'practice', 'reinforcement' and 'revision'. Discussion often linked these broader pedagogical themes to more specific operational themes.

The theme of *Investigation promoted* not only associates ICT with an 'investigative' approach to developing mathematical ideas, but suggests that technology may play an important part in promoting such an approach:

Well we're still compelled to use it really if we're going to teach it in an investigative way. . . / It's a much more investigative approach, because of the ICT facilities that you got. [VC]

They can do more investigative work, can't they. If something doesn't work, then they can try something else. / You can make it more open-ended as well. You can make tasks more open ended. [GS]

As the latter quotation suggests, the earlier theme of *Tinkering assisted* indicates one way in which ICT use supports an investigative approach. Further quotations also invoke facets of *Restraints alleviated* and *Routine effected*, and then *Features accentuated* and *Attention raised:*

They're carrying out investigations that it wouldn't be sensible to do with pencil and paper and would just be tedious and repetitious to do with an ordinary calculator. [SC]

So if you want to graph y equals two x, you can just put it in and it draws the graph for you, and then you can look at how much it slopes and they can see what happens to it as you change y equals three x, y equals four x, and it is just very neat for any set, but bottom sets in particular who can't draw graphs very well, because you can concentrate on the analysis. / Otherwise you are drawing it on the board for them and they are just sitting watching, whereas they are doing something, they are looking for themselves. [LC]

The theme of *Consolidation supported* associates ICT use with the 'practice', 'reinforcement', 'revision' of mathematical knowledge and skills. Teachers often – but not invariably – referred to courseware – notably SMILE – designed expressly for this purpose:

We've got the SMILE package which worked very well with Year 7 to reinforce basic skills. [GS]

And what we did was put the relevant SMILEs into the module that we were doing at the time, and they played they games relevant to, more as a reinforcing and practising rather than learning. [VC]

Essentially, all that [Angle 360] does is it offers you. . . a diagram of an angle. It gives you an angle size. . . and as one arm goes round, you have to stop it when it has opened out to the angle size specified, and then it tells you whether you are a good boy or a bad boy and how close you are. And it is very very simple. But the teaching points are still valid. [LC]

As well as valuing such courseware, teachers reported positive experiences of using an interactive website designed to help students revise for national examinations:

In Year 11, between the mocks and their actual exams, we did work on GCSE BiteSize on the Internet, which we just took them down, showed them how to get into the site, and just let them work at their own pace, so they could pick which topics they wanted to work from. They could do a revision bit on there, then they could actually do a test and then go back so it was almost like self-study, but with us in the room to help them. And they really enjoyed that, they got on well and I know quite a few of them then went on in their own time and looked at it, as well. [MC]

Moreover, teachers also noted how certain software could be effective in supporting both consolidation and investigation, if tasks were suitably tailored:

The Omnigraph [graphing package] I've used for a revision package for GCSE students looking at different functions where they have a graph drawn and they have to guess what the function is by drawing their own to make it superimposed over it. It's been quite a good revision package. It's also a good way of investigating it in the first place. [VC]

So when we have used Coypu [graphing package]... it is because... we are either preparing them to learn about gradients and doing straight line graphs, or we have already done it so they can actually see "Oh this is what we did in the classroom", and then practice lots and lots more of them than they ever would if they had to do it with pen and paper. [LC]

Incidence of pedagogical themes and co-incidence with operational themes

Again, a quantitative analysis provides an overview of relationships between these pedagogical themes and the operational themes discussed earlier. The key indicators are assembled in Table VI. A row of summary data for each of the two pedagogical themes shows first the number of transcript sections containing text units coded to that theme, and then the proportion of these sections containing text units coded to each of the operational themes.

Bearing in mind the relatively small numbers of transcript sections involved, these profiles show only a few important points of difference: in associating *Routine facilitated* and *Attention raised* exclusively, and *Restraints alleviated* more prominently, with *Investigation promoted.* This is consistent with what was noted earlier: that while teachers reported computer use as supporting both consolidation and investigation, its contribution to the latter tended to be seen as more crucial, by making investigative activities accessible to students, even viable in the classroom.

Distribution of themes across departments

Although this paper is concerned with the substance of ideas rather than their provenance, it is important to gain some sense of the distribution of particular clusters of ideas. Relevant summary data are assembled in Table VII. Each row of the table reports on a particular school, showing

TABLE VI

Incidence of pedagogical themes and co-incidence with operational themes

Pedagogical theme	Number of sections coded to theme	Proportion of these sections coded to operational themes									
		Ambience enhanced	Restraints alleviated	Tinkering assisted	Motivation improved	Engagement intensified	Routine facilitated	Activity effected	Attention raised	Features accentuated	Ideas established
Investigation promoted	12	8%	25%	17%	17%	17%	25%	8%	17%	17%	8%
Consolidation supported	16	6%	13%	13%	25%	13%	0%	6%	0%	19%	13%

which of the operational and pedagogical themes occurred in the relevant interview transcript. From this, it is clear that all of the themes were invoked in at least five schools, and most in at least six, indicating wide currency. The two atypical school profiles are those of Community College at one extreme and Technology College at another. At Community College, the operational emphasis is on issues of participation, and the pedagogical emphasis on consolidation. This is consistent with two trends that have been noted elsewhere in less socially advantaged schools, and in the early stages of ICT use. Equally, the other outlier is Technology College -a department drawing on more longstanding experience, and with easily the most intensive use- and here the pattern is largely the reverse, with little or no mention of issues of participation and consolidation; rather an emphasis on processes supporting pace, productivity and progression, and a pedagogical focus on investigation.

SYNOPSIS OF FINDINGS AND SYNTHESIS WITH PRIOR RESEARCH

Review of the practitioner model

This study has elicited from practitioners a model of the successful use of ICT to support classroom teaching and learning in mathematics. The model is organised as a system of themes identifying key processes and critical states. Four themes depend most directly on exploiting affordances of ICT: *Ambience enhanced* in changing the general form and feel of classroom activity; *Tinkering assisted* in helping to correct errors and experiment with possibilities in carrying out tasks; *Routine facilitated* in enabling subordinate tasks to be carried out easily, rapidly and reliably; and *Features accentuated* in providing vivid images and striking effects which highlight properties and relations. Three further themes depend in turn on these processes: *Restraints alleviated* in mitigating factors inhibiting student participation such as the laboriousness of tasks, the requirement for – and the demands imposed by – pencil-and-paper presentation, and vulnerability to mistakes being exposed; *Motivation improved* in generating student enjoyment and interest, and building student confidence; and *Attention raised* in creating the conditions for students to focus on overarching issues. Three final themes depend again on preceding processes: *Engagement intensified* in securing the commitment, persistence and initiative of students in classroom activity; *Activity effected* in maintaining the pace and productivity of students within classroom activity; *Ideas established* in supporting the development of student understanding and capability through classroom activity.

TABLE VII

Occurrence of operational and pedagogical themes by school

School [Code]	Themes occurring											
	Ambience enhanced	Restraints alleviated	Tinkering assisted	Motivation improved	Engagement intensified	Routine facilitated	Activity effected	Attention raised	Features accentuated	Ideas established	Investigation promoted	Consolidation supported
Community College [CC]	✓	✓	✓	✓	✓							✓
Girls School [GS]	✓	✓	✓	✓		✓	✓	✓	✓	✓	✓	✓
Language College [LC]	✓	✓	✓	✓	✓	✓	✓	✓	✓	✓	✓	✓
Media College [MC]	✓	✓		✓	✓	✓	✓	✓	✓	✓	✓	✓
Sports College [SC]		✓	✓	✓	✓	✓	✓	✓	✓	✓	✓	✓
Technology College [TC]	✓			✓		✓	✓	✓	✓	✓	✓	
Village College [VC]	✓	✓	✓	✓	✓	✓	✓	✓	✓	✓	✓	✓
Overall incidence	6	6	5	7	5	6	6	6	6	6	6	6

This model serves as a helpful abstraction, providing a generic scheme capturing the major elements of practitioner conceptualisation of the use of ICT to support classroom teaching and learning in mathematics. In any actual instance of classroom activity, of course, certain components of the system are likely to assume more prominence than others, and those operative components of the scheme will be filled out more concretely according to the particular circumstances. The examples provided in the earlier discussion offer illustrations of both these aspects. More generally, while many of the themes are equally relevant to classroom activity whether conceived in terms of investigation or consolidation, the cluster of themes focusing on *Routine facilitated, Restraints alleviated* and *Attention raised* are particularly relevant to investigative tasks. The model is not a deterministic one. Rather it highlights key processes and critical states which require active – and reactive – planning and management on the part of the teacher for ICT use to successfully support teaching and learning.

The model is a tentative one. It is based only on teachers' somewhat decontextualised accounts of what they saw as successful practice, elicited through group interview, rather than on more strongly contextualised accounts of specific instances of practice, supported by examination of actual classroom events. Nevertheless, in subsequent development work, earlier variants of this model -illustrated by actual accounts of specific classroom examples- have helped teachers to articulate their conceptions of how particular forms of ICT use support teaching and learning. These 'practical theories' can then undergo testing and refinement as teachers focus their attention on this aspect of their professional practice. Likewise, ideas reported within the wider professional and research literatures may stimulate teachers to generate more ambitious 'practical theories' of ICT use, and to seek to realise them in – and adapt them to – more mainstream circumstances of teaching and learning. Equally, the present model is grounded in particular educational circumstances – those of the contemporary English school system and national curriculum, and in national terms, of relatively socially privileged and academically successful state schools in the Cambridge area. All of these comments point to ways in which the model can usefully be further tested, refined and extended.

Synthesis with prior research

However, there are sufficient continuities between these British findings and those of the American case studies reviewed earlier, to suggest that the model – or at least important elements of it – may be more widely transferable. As reported, the teacher in the Myhre study had a rationale for computer use which was primarily concerned with student participation and productivity, placing particular emphasis on aspects of *Ambience enhanced* and *Restraints alleviated* – changing the classroom environment by moving away from work through the medium of pencil and paper; *Tinkering assisted* – creating opportunity for students to try out different answers to problems; *Motivation improved* – building student confidence and enjoyment through such work; and *Activity effected* – increasing the number of problems completed by students over the course of a lesson. Correspondingly, the teaching approach described in the Doerr and Zangor study was primarily concerned with student participation and progression, by capitalising on the affordances of calculator use to support aspects of *Tinkering assisted* – in checking results and trialling conjectures; of *Routine facilitated, Restraints alleviated* and *Attention raised* – in effecting routine aspects of calculating, graphing and data processing, removing the tedium of doing so, and shifting the focus of attention towards interpretive issues; and of *Features accentuated* – in providing visual images; so as to promote various forms of *Ideas established*. In addition, one further idea emerges more strongly from this study, having been raised only marginally within the English accounts. What might be termed *Alternatives enabled* associates the use of ICT with providing variant mathematical strategies, contrasting with those normally employed without ICT – such as solving equations graphically or numerically rather than analytically. This points to valuable ways in which the model might be extended and refined through further studies in a wider range of contexts.

On the other hand, the findings of this study indicate limitations, at least in the British context, of conceptualising the use of ICT in mathematics teaching and learning in terms of an opposition between 'constructivist' and 'didactic' (or 'transmission') pedagogies, and of classifying software in similar terms. First, for the teachers in this study, investigation and consolidation appeared to be complementary aspects of teaching and learning; the former concerned with opening up new ideas, the latter with securing them. Second, certain types of software, and even certain types of task, were claimed to be of value for both purposes; what changed their functioning was the prior knowledge and learning expectations that teacher and students brought to them. However, this way of looking at teaching and learning may be a British idiosyncrasy; as the particular discourse of 'investigation' certainly is.

Perhaps, however, it is the ideas of participation, pace and productivity, and progression as indices of successful classroom activity, and the view of ICT use as successful in as far as it contributes to these, which are more widely transferable. This would certainly accord with suggestions that schooling has its own deep structure centred on curricular coverage and class instruction (Hamilton, 1989), and that this 'DNA of classroom life' determines in large part the form of classroom teaching and learning, inhibiting the uptake of successive generations of new information and communication technologies, and shaping such use as does occur so as to fit established practice (Cuban, 1986). Nowadays, as immediate constraints – such as the limited availability of technology in schools – are diminishing, a more fundamental gap is emerging between the conceptions of teaching, learning and thinking guiding the development of pioneering computer-based learning environments, and those underpinning the traditional professional knowledge of teachers, contributing to the slow and uneven pace with which such technologies are being incorporated into everyday practices of mathematics education (Balacheff and Kaput, 1996).

What this study hints at is a gradual mechanism whereby teachers initially view technology through the lens of their established practice, and employ it accordingly. Even at this stage, however, shifts in practice may result. Most prominently here, teachers suggested that, by helping to create the classroom conditions in which investigations could be conducted successfully, particularly with lower attaining pupils, use of technology made this form of classroom activity a more viable option. So while, in one sense, such use of technology was simply assisting teachers to realise an established form of practice, what is significant is that it was enabling them to employ this practice more effectively and extensively. At the same time, however, this and other ways of using technology were giving rise to unanticipated phenomena, such as tinkering by students, which were leading teachers to start to reconsider aspects of their practice. This resonates with a pattern of 'cautious adoption' (Kerr, 1991) in which technology figures in teachers' thinking about their practice as a contributory, but subsidiary, factor. As well as serving as a 'lever' through which teachers seek to make established practice more effective, technology appears also to act as a 'fulcrum' for some degree of reorientation of practice, and a measured development of teachers' pedagogical thinking.

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NOTES

1. Advanced calculators are – in effect – a form of hand-held computer with built-in mathematical software. However, they may be overlooked in studies of 'computer use', leading to underestimation of its extent in mathematics classes
2. In recent years 'Information and Communication Technology' (ICT) has started to displace 'Information Technology' (IT) as the accepted collective term for computer-based tools and resources across the English education system.
3. The simple measures employed here aim to describe patterns in proportional terms. The underlying situation does not conform to standard assumptions for statistical inference – notably those of independent trials – and this raises epistemological objections to employing statistically conventional measures of association between categories – such as the Yule-Boas phi coefficient or the related correlation and chi-squared coefficients (Guilford, 1965). In particular, because such coefficients would take as confirmatory of association between two themes those sections of transcript in which neither occurs, their use would raise one of the well known confirmation paradoxes (Salmon, 1995). The coefficient of coincidence used here is – as it happens – equivalent to a limiting version of the Yule-Boas phi coefficient from which reference to such sections has been eliminated.

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