Forthcoming *Educational Studies in Mathematics*

Using Graphing Software to Teach about Algebraic Forms: A Study of Technology-Supported Practice in Secondary-School Mathematics

Kenneth Ruthven, Rosemary Deaney & Sara Hennessy
University of Cambridge, Faculty of Education

Abstract: From preliminary analysis of teacher-nominated examples of successful technology-supported practice in secondary-school mathematics, the use of graphing software to teach about algebraic forms was identified as being an important archetype. Employing evidence from lesson observation and teacher interview, such practice was investigated in greater depth through case study of two teachers each teaching two lessons of this type. The practitioner model developed in earlier research (Ruthven & Hennessy, 2002; 2003) provided a framework for synthesising teacher thinking about the contribution of graphing software. Further analysis highlighted the crucial part played by teacher prestructuring and shaping of technology-and-task-mediated student activity in realising the ideals of the practitioner model. Although teachers consider graphing software very accessible, successful classroom use still depends on their inducting students into using it for mathematical purposes, providing suitably prestructured lesson tasks, prompting strategic use of the software by students, and supporting mathematical interpretation of the results. Accordingly, this study has illustrated how, in the course of appropriating the technology, teachers adapt their classroom practice and develop their craft knowledge: particularly by establishing a coherent resource system that effectively incorporates the software; by adapting activity formats to exploit new interactive possibilities; by extending curriculum scripts to provide for proactive structuring and responsive shaping of activity; and by reworking lesson agendas to take advantage of the new time economy.

Keywords: classroom teaching practice; computer graphing software; secondary-school mathematics; teacher knowledge and thinking; technology use and integration.

1. Introduction

This paper develops a line of research on the incorporation of computer-based tools and resources into the mainstream practice of mathematics teaching in secondary schools. This line began by building a generic model of what teachers regard as successful practice (Ruthven & Hennessy, 2002; 2003), drawing on decontextualised accounts elicited through group interviews with mathematics departments. A subsequent project has sought out and examined professionally well-regarded mainstream practice in secondary mathematics and science teaching, drawing on more strongly contextualised accounts of specific instances of particular types of technology-supported practice, supported by examination of actual classroom events. This paper reports a study of teaching practice incorporating use of graphing software; another study has examined dynamic geometry (Ruthven, Hennessy & Deaney, 2005; 2008).

2. A practitioner model of the contribution of technology use to teaching mathematics

The earlier research addressed the broad question of how teachers – specifically mathematics teachers in English secondary schools – conceive the incorporation of computer-based tools and resources into their classroom lessons. Drawing on teachers' accounts of successful practice, this work developed a 'practitioner model' (Ruthven & Hennessy, 2002) which was refined into a more compact form (Ruthven & Hennessy, 2003) in the light of cross-subject comparison (Ruthven, Hennessy & Brindley, 2004). The themes in this model highlight the contribution of digital tools and resources to:

- *Effecting working processes and improving production,* notably by increasing the speed and efficiency of such processes, and improving the accuracy and presentation of results, so contributing to the pace and productivity of lessons;

- *Supporting processes of checking, trialling and refinement,* notably with respect to checking and correcting elements of work, and testing and improving problem strategies and solutions;
- *Overcoming pupil difficulties and building assurance,* notably by circumventing problems experienced by pupils when writing and drawing by hand, and easing correction of mistakes, so enhancing pupils' sense of capability in their work;
- *Focusing on overarching issues and accentuating important features,* notably by effecting subsidiary tasks to support attention to prime issues, and facilitating the clear organisation and vivid presentation of material;
- *Enhancing the variety and appeal of classroom activity,* notably by varying the format of lessons and altering their ambience by introducing elements of play, fun and excitement and reducing the laboriousness of tasks;
- *Fostering pupil independence and peer exchange,* notably by providing opportunities for pupils to exercise greater autonomy and responsibility, and to share expertise and provide mutual support.

Subsequent research has offered collateral support for the model. An independent Paris study (Caliskan-Dedeoglu, 2006; Lagrange & Caliskan-Dedeoglu, in press) found that the original version of the model provided a useful template to describe teachers' pedagogical rationales for the classroom use of dynamic geometry. However, when teachers were followed into the classroom it became clear that these rationales sometimes proved difficult to realise in the lessons themselves. Teachers could be overly optimistic about the ease with which students would be able to use the software. Far from effecting working processes and overcoming pupil difficulties, computer mediation might actually impede student actions, with the teacher trying to retrieve the situation by acting primarily as a technical assistant. Likewise, students could encounter difficulties in relating the figure on the computer screen to its paper-and-pencil counterpart. Rather than accentuating important features, computer mediation might actually mask them. This serves to emphasise that while the model identifies the types of 'normal desirable state' which teachers associate with successful technology use, actually achieving such success depends on creating classroom conditions and pursuing teaching actions which establish and maintain these states (Brown & McIntyre, 1993). Certainly in a parallel Cambridge study of classroom practice in using dynamic geometry, teachers were found to have developed strategies to anticipate, avoid and overcome such potential obstacles (Ruthven et al. 2005). What these studies emphasise, then, is that the model represents a guiding ideal for teachers. To be able to realise this ideal in practice depends on teachers developing craft knowledge to underpin their desired classroom use of new technologies.

The term 'craft knowledge' refers to the largely reflex system of situated expertise which teachers develop, tailored to their professional role and embedded in their classroom practice (Brown & McIntyre, 1993; Leinhardt, 1988). Compared to the more decontextualised and rationalistic approach to characterising a 'professional knowledge base for teaching' in which the 'representations of [subject] knowledge in teaching' associated with 'pedagogical content knowledge' are highlighted (Wilson, Shulman & Richert, 1987), the craft perspective focuses on the functional organisation of a broader range of teacher knowledge to accomplish concrete professional tasks. In particular, it recognises that much innovation calls for adaptation of craft knowledge with respect to key structuring features of classroom practice such as working environment, resource system, activity format, curriculum script, and time economy (Ruthven, 2007; in press). The use of new technologies often involves changes in the *working environment* of lessons in terms of room location, physical layout, and class organisation (Jenson & Rose, 2006), requiring modification of the *classroom routines* which enable lessons to flow smoothly (Leinhardt, Weidman & Hammond, 1987). Equally, while new technologies broaden the range of tools and resources available to support school mathematics, they present the challenge of building a coherent *resource system* (Amarel, 1983) of compatible elements that function in a complementary manner and which participants are capable of using effectively. Likewise, innovation may call for adaptation of the established repertoire of *activity formats* that frame the action and interaction of participants during particular types of classroom episode (Burns & Anderson, 1987; Burns & Lash, 1986), and combine to create prototypical *activity structures* or *cycles* for particular styles of lesson (Monaghan, 2004). Moreover, incorporating new tools and resources into lessons requires teachers to develop their *curriculum script* for a mathematical topic. This 'script' – in the psychological sense – is an event-structured organisation of variant expectancies for the lesson and of alternative courses of action, forming a loosely ordered model of goals, resources and actions for teaching the topic (Leinhardt, Putnam, Stein & Baxter, 1991); it interweaves mathematical ideas to be developed, appropriate topic-related tasks to be undertaken, suitable activity formats to be used, and potential student difficulties to be anticipated, guiding the teacher in formulating a particular *lesson agenda,* and in enacting it in a flexible and responsive way. Finally, teachers operate within a *time economy* in which they seek to improve the 'rate' at which the physical time available for classroom activity is converted into a 'didactic time' measured in terms of the advance of knowledge (Assude, 2005).

3. Research on teaching practices incorporating use of graphing technology

Software for mathematical graphing on the coordinate plane was amongst the earliest computer applications developed for professional and educational use. From the start, mathematics educators have seen such software as having more than just a pragmatic, computation-effecting function in translating symbolic expressions (such as coordinates and equations) into graphic images (such as points, lines and curves); but as also having an epistemic, concept-building function, through supporting exploration of symbolic-graphic relationships. In an early survey, Fey (1989, p. 247) noted that "suggestions of ways to use such graphs [as pictures of algebra] have appeared in many places and the software tools available to facilitate graphing are really quite versatile and easy to use". He indicated that, for teaching purposes, graphing software was typically first used to display series of graphs as a means of revealing the patterns associated with various rule types and the significance of parameters within each type. He reported that it was then common to ask students to generate rules to match a given graph or to fit given points; here Fey referred to the success of a piece of educational software which placed such tasks within a game format.

Over the course of the 1990s, several pioneering studies investigated aspects of teacher thinking and classroom practice in using graphic calculators at upper secondary level. The study with concerns most similar to those of this paper was conducted by Simmt (1997) who examined the beliefs and practices of teachers introducing the use of graphing calculators into a unit on quadratic functions. The machines were used primarily to generate patterns of graphic images; most commonly by students themselves; and in the majority of cases through a guided-discovery approach which some teachers reported as made possible only through use of the technology. The reasons that the teachers gave for using graphing technology related principally to saving lesson time and generating more examples, and to increasing instructional variety and enhancing student motivation, corresponding respectively to the *Effecting working processes and improving production* and *Enhancing the variety and appeal of classroom activity* themes of the practitioner model. In the majority of cases, teachers considered the calculator valuable as a tool for checking students' sketches or manipulations, relating to the theme of *Supporting processes of checking* (but apparently with little *trialling and refinement* involved). One teacher mentioned aspects of *Overcoming pupil difficulties and building assurance* in reporting that using the calculator increased students' confidence in the accuracy of their graphs, and that this enabled them to work with less dependence on the teacher, in line with *Fostering pupil independence* (but without reference to *peer exchange*). Finally, some teachers expressed concern about limitations of calculator graphing in respect of the frequent need to adjust the graphing window and to interpret approximate coordinate values, so qualifying the perceived contribution of this technology to an ideal of *Focusing on overarching issues and accentuating important features*.

Other studies throw more oblique light on the concerns of this paper. Farrell (1996) studied the classroom practice of teachers nearing the end of their first year of involvement in a development project in which they were working with teaching materials to which use of graphing technology was integral. The study concluded – with appropriate caution – that technology use helped to create conditions under which it became possible – but not inevitable – for formats for classroom activity to shift towards less teacher exposition and more student investigation and group work, providing greater scope for both teacher and students to take on explainer, consultant, and co-investigator roles. Doerr & Zangor (2000) studied the practice of a teacher who was very experienced in graphing calculator use. The main findings relate to typical modes of classroom calculator use, as modelled and encouraged by the teacher, and grounded in her broader instructional approach: as computational tool, transformational tool, data collection and analysis tool, visualizing tool, and checking tool. Although this study has a strong pedagogical focus, the idea of graphing calculators as media for developing and deploying variant mathematical strategies also emerges as important: strategies differing markedly from those already recognised, and strongly dependent on distinctive affordances of the new tools, such as solving equations graphically or numerically rather than analytically. This idea is absent from, or marginal in, the other studies discussed here (for example, being generally rejected by the teachers participating in Simmt's study).

Over the last few years, the use of graphing technology has become more widespread and commonplace, notably moving down from relatively advanced courses to earlier levels of secondary education. However, the teacher thinking and classroom practice associated with wider use appear to have received little attention from researchers. Moreover, few studies at any level have been conducted outside North America, and few have examined the use of graphing software as against graphic calculators. The study to be reported in this paper addresses all these aspects of underinvestigation in the field. It complements Godwin & Sutherland's (2004) study which compared two lessons taught by teachers involved in an initiative to develop classroom use of new technologies. In one lesson, graphic calculators supported work on straight line graphs; in the other graphing software provided a means of studying quadratic graphs. Describing the unfolding teaching sequences in each lesson, Godwin & Sutherland's paper emphasises the way in which teachers played "a vital role in orchestrating and structuring classroom activities in such a way as to support students to focus attention on appropriate mathematical ideas" (p. 132) and the way in which this creates "an emergent and collective… community… in which knowledge construction converges to some acceptable 'common knowledge'" (p. 150). In particular, they note how, in both lessons, teachers introduced critical attributes of the graph types through considering a succession of parameterised sub-families, with the graphing technologies used to give students the power to experiment with many members of each sub-family within a relatively short period of time. Our study complements this work by incorporating a stronger teacher perspective into the overall analysis of technology-supported classroom practice, and by extending analysis of the specific handling of graph types to bring out the interaction between technological affordances, task designs, and teaching actions.

4. Design and method for the study

In the first phase of the larger research project from which this study derives, recommendations from professional leaders and evaluations from school inspections were triangulated in order to identify subject departments in state-maintained schools that were regarded as successful both in terms of the general quality of their practice and their use of computer-based tools and resources within it. To ascertain what practitioners themselves regarded as successful practice, focus group interviews were then conducted with each of these subject departments (during the latter part of the 2002/03 school year). In these interviews teachers were invited to nominate and describe examples of successful practice involving use of computer-based tools and resources. Through this process, several practices were selected for more intensive investigation in the second phase of the research through conducting case studies (during the 2003/04 school year).

From the focus group interviews with eleven mathematics departments, use of graphing technology was identified as a successful established practice, favourably mentioned in all the departments, and selected as a nominated example in seven, with graphing software generally preferred over graphic calculators. However, outside advanced courses, use of this technology still proved relatively infrequent, occurring on no more than a handful of occasions per school year with any particular class. The particular mathematical topics which were most frequently cited were linear equations (nominated in six departments, and mentioned in four more) and quadratic equations (nominated in only one department, but mentioned in four more, all of which nominated linear equations). The use of graphing technology to treat such equations also featured prominently in official curricular and pedagogical guidance (DfEE, 2001; DfES, 2003), and lessons of this type were cited favourably, both in reports of individual school inspections, and in a national report on the impact on secondary mathematics of government ICT initiatives in schools (OfStEd, 2004: p. 7). A further index of the archetypicality of this practice was provided when one department (which had nominated it as an example) volunteered that for their forthcoming professional development session on ICT in mathematics teaching they had “specifically asked for it to be *not* drawing graphs and straight lines”!

Available project resources made it feasible to follow up only a limited number of cases of each practice. Given that there were relatively few differences in the forms of graphing practice that teachers described, and that graphing software (rather than graphic calculator) was clearly teachers' preferred technology, we followed up two cases of this type which provided some degree of contrast in approach, located in schools in different regions of England. Other important considerations were that the teachers concerned had already provided quite full and thoughtful accounts during the focus group interviews, and that two lessons could be observed with each teacher, one on linear forms and the other on quadratic. Detailed observation records were made of each lesson, incorporating a transcript of the main episodes, integrated with further observational material including copies of other resources used and records of the graphs displayed. Post-lesson interviews were conducted with teachers after each observed session. These were organised around a standard sequence of printed cards asking teachers about their thoughts, first while preparing the lesson (what they wanted pupils to learn; how they expected use of the technology to help pupil learning); then looking back on the lesson (how well pupils learned; how well the technology helped pupil learning; important things that they were giving attention to and doing). At the end of the first interview, teachers were also asked about any pitfalls that they had experienced in using graphing technology. At the end of the second interview, they were invited to suggest any ways in which their approach differed between the two lessons. These interviews were audiotaped and transcribed.

The subsequent process of analysis was in three stages. First, the file for each lesson was analysed in the following ways, providing the basis for assembling a summary of the lesson, focusing specifically on the teaching practice and practitioner thinking associated with it. From the observation record, occasionally amplified by illuminating material from the interview transcript, a basic outline of the working environment, resource system, lesson agenda, and activity structure of the lesson was compiled. From the interview transcript, occasionally amplified by illuminating material from the observation record, a narrative was constructed to set out the main lines of thinking that teachers reported as lying behind the lesson and emerging in response to it. Here, to provide a common core of organising constructs, the broad themes from the compact version of the earlier ‘practitioner model’ (Ruthven & Hennessy, 2003) were used as appropriate, but the narratives themselves drew directly on the source material, going beyond the earlier themes where necessary (Strauss & Corbin, 1994). These lesson summaries are presented in Boxes 1 to 4, and sketch the classroom practice observed in each lesson and the teacher thinking associated with it (with the indexing of segments used to facilitate later cross referencing from this main text). Second, an analysis was conducted across lessons and teachers, again organised in terms of the earlier practitioner themes and elaborating them where necessary, to produce a model of this technology-supported teaching practice, grounded in the teacher accounts and teacher actions captured in the evidence base. Third, sensitised by the earlier literature review and by wider theory in the field to aspects of the teaching practice which remained largely tacit in this model, some further key issues –concerned with instrumental induction, task design, teacher intervention, and craft knowledge – were examined, drawing more reflectively on the case files to extend the earlier analysis.

Box 1: Lesson summary A1 – Investigating linear forms

[1] This one-hour lesson involved a Year 10 (Grade 9) class of lower-attaining students. Students were seated individually at a machine (after initial difficulties in finding usable ones) and a computer was available to the teacher, but with its projector not functioning at the start of the lesson.

[2] The resource system for the lesson consisted of a teacher-devised sequence of worksheets involving use of graphing software. The absence of the projector forced the teacher towards less efficient methods of guiding students how to access the software and introducing the techniques needed for the lesson:

> Not having it there meant that I had to go round much more on an individual basis and say "Right, you need to grab that hand if you want to pull the graph down." If I'd had the projector there, it would have been much easier to show the whole class at once.

[3] The lesson agenda followed the sequence of worksheet tasks. Each task involved using the software to generate specified members of a family of graphs (of the forms $y = c$, $x = c$, $y = x + c$, $y = mx$; for example $y = x + 2$, $y = x + 4$, $y = x - 1$, $y = x - 3$), recording these graphs on the worksheet, then completing a statement or answering a question designed to highlight their generic characteristics.

[4] Over most of the lesson, the activity format alternated between short bursts of whole-class, teacher-led exposition and questioning to quickly prefigure or review a worksheet task, and more extended periods of individual student work on these tasks with the teacher circulating to provide guidance and support. This formed an activity cycle covering each task in turn. The lesson concluded with a 15-minute plenary, reviewing and elaborating findings through teacher-led presentation and questioning.

[5] In preparing the lesson, a prime concern of the teacher had been to secure the active engagement of students. The interactive character of work with a computer and the requirement to record results on the worksheet were both intended to help achieve this:

> I wanted something that was going to engage the pupils actively, so I gave them a worksheet to write on, so that they could actually all have something to do, none of them would be sitting there not doing anything. Because they're quite a low ability group, and some of them lack motivation. So I was trying to think of a way to keep them engaged.… I usually find that the computer room will motivate pupils' learning, because they'll want to see what the computer does next. And that also engages them, because they've got to actually press something and see what happens.

[6] Equally, using the software made tasks more accessible and less laborious to students, and gave them the satisfaction of getting more immediate results:

> If they'd had to just draw that, without having the computer there to help them, some of them would have really struggled.… Also some of them would have just completely turned off if they'd had to draw out a table of values for each graph. It would have taken them a long time and been quite laborious, whereas the technology just meant they could fly through it and get an instant result, and I think they enjoyed that.

[7] Likewise, inviting students to test out their proposed solution to a problem by entering it into the projected computer contributed to engagement and enjoyment:

> That really helped some of the students in terms of their motivation and their learning, because they came out and they wanted to get the right thing in front of the whole class, and they could actually see it come up on the huge board at the front, which some of them really enjoyed. And that was good fun, and helped them to see what they were doing.

[8] Using the software to effect the production of graphs reduced the time required to undertake the lesson:

> It's much quicker. I mean, if you'd done the same lesson in a classroom situation, it probably would have taken one lesson to do $y = 5$… and $x = 2$ etcetera. And then it would have been another lesson to start on diagonal lines.

[9] The teacher noted the striking way in which altering a coefficient modifies the computer graph, and how this helps to make properties of linear graphs accessible to students:

> They can see each time you add a different number to x it cuts the y axis at a different point, and they can see that straight away.

[10] The teacher also identified characteristics of graphing software which aid interpretation of graphs: the colour coding which associates an equation with its graph; and gridlines which make the size of a gradient more salient:

> Because each graph's a different colour… they can see which one goes with which.
>
> And [the graph] is really clear with gridlines behind it. They can see that the gradient of the graph changes if you change the number before x, and it's just so clear and easy for them to see.

[11] Equally, the teacher reported a key move intended to guard against students not appreciating the graph as a set of points satisfying the corresponding equation:

> With the first line they drew up, I asked them to give me some co-ordinates of any point on that line, so they could see the co-ordinates as well. I only did that for the first question, because I then think that they'd got the idea of giving the co-ordinates, which gives them some mathematical interpretation.

[12] This formed part of a more general pattern of teacher structuring of student work, and questioning during it, to maintain lesson pace and support learning progression:

> I tried to keep the pace going and to vary the task slightly every few minutes, so that they were looking for something extra, and asking key questions to make sure that they were getting something useful out of the graphs they were drawing.

[13] The teacher used the software to support mathematical experimentation. Following a review of graphs of the form $y = x + c$, she posed a question:

> I said "How would you draw the diagonal line going the other way across the graph? So going from top left to bottom right?" And… there were several different things called out and we could try them very, very quickly, and check them.

[14] She noted how the computer enabled students in their own exploration:

> Some of them also went on to explore for themselves, and try out, using their technology, which again is something, if you didn't have the technology, you just could not do.

She describing supporting one such example:

> He'd drawn an equation which was $x = -yx$, and… he noticed that that was the same as $y = -1$, and he wondered why, so we then rearranged and simplified that equation.

[15] Likewise, commenting on how she would have started the lesson had the projector been available, the teacher talked of following her exposition of basic technique with some playful exploration intended to expose students to a wider range of features:

> And then I would have said, "You've got loads of different buttons on here", and I think some of them would have wanted to know what some of the extra buttons were. And we probably would have had five minutes having a kind of play, getting the feel of it. Just to show them some of the other features.

Box 2: Lesson summary A2 – Investigating reciprocal and quadratic forms

[1] This one-hour lesson involved a Year 9 (Grade 8) class of higher-attaining students. It took place in a computer suite with students seated individually at a machine and a projectable computer available to the teacher. Technical difficulties prevented students printing out their work towards the end of the lesson.

[2] The resource system for the lesson consisted of two textbook 'investigation and discussion exercises' involving use of graphing software. The class was already familiar with the software, having used it several times, but in the opening review the teacher reminded students where "to find the equation button" in case "some of you have forgotten where that is".

[3] The lesson agenda consisted of a quick opening review of linear graphs, followed by work on the structured investigations in textbook order. The first started by asking about the range of x and y values featuring in the graph of $y = 1/x$, and about ranges of x values for which y was increasing or decreasing; it then named groups of specific reciprocal functions and asked students to explore relationships between the graphs in each group (such as $y = 1/x$, $y = 2/x$, $y = 3/x$, $y = 4/x$). The second investigation specified four groups of quadratic functions (such as $y = x^2$, $y = x^2 + 2$, $y = x^2 - 2$, $y = (x + 2)^2$, $y = (x - 2)^2$) and suggested examining cuts on the axes, extreme values, and lines of symmetry, to explore what the graphs in each group had in common.

[4] The activity format within the lesson alternated between periods of up to 10 minutes of whole-class, teacher-led exposition and questioning to prefigure, elaborate or review an investigation task, and periods of individual student work on these tasks with the teacher circulating to provide guidance and support:

> Today was more of them investigating, exploring, almost on their own, but bringing it all back to a whole class thing every so often.

[5] In preparing the lesson, a prime concern of the teacher had been to stretch the class:

> It was an able group, so I wanted something that would stretch them and take them on a step further.… [So] we went on to looking at reciprocal graphs… looking at what happens as x is increasing… That was quite challenging for most of them. But the technology helped.

[6] While the teacher could expect students in this class to apply themselves, she reported that exploring with the computer particularly motivated them:

> I just said "Come in and go onto Autograph" and… two seconds later they were in, sat at a computer, logged on onto Autograph.… And they all wanted to explore and they were really enthused and excited.

[7] She identified students who did written work only reluctantly as achieving more through this type of computer-supported investigation:

> [Named students] are both pupils that aren't particularly well motivated in written work, and I think using the computers was really successful for them, and I think they achieved a lot more than they often would in a classroom situation.

[8] Using the graphing software helped impart pace to the lesson by expediting routine tabulation and graphing:

> All of these things were done so much more quickly, because the technology was there to help them. These pupils are able pupils, they can draw a table of values and draw the graph. They've done that many times, and the technology's just helping them to move through everything at a much quicker pace.

[9] She noted how one feature of the software interface was particularly helpful in bringing out properties of the reciprocal graph: using dragging to follow the limiting trend of the graph:

> [The technology] was really useful today with the reciprocal graph. We were looking at, as x decreases, what happens to y. And they could use the hand to grasp the graph down, and look further up the graph to see what y tended to.… Obviously if they'd drawn it, they would had had to have plotted so many values, and it would have been quite difficult to find a suitable scale for the axes. So I think the technology just made it a lot quicker and easier to see what was happening.

[10] The teacher questioned students to check their understanding of the relationship between equation and graph, introducing consideration of the coordinate values of particular points where necessary:

> I was going round really checking that pupils understood what the graph showed, and relating it back to the equation. So if they didn't understand why the computer drew a certain graph, I'd then go back to a table of values with them… showing them where that would be plotted on the graph.

[11] Equally, the teacher questioned students to deepen their understanding of the limiting trends, and prompting them to exploit the software accordingly:

> I also asked them key questions, like "What happens as x decreases? What's happening to y?" And some of them would just say [things like] "It looks like it's getting bigger but I don't know why or what that means." And so then we'd look at it in a little bit more detail. And zoom in on the graph and see what was actually happening.

[12] The teacher noted how the graphing software opened up possibilities of experimenting with a less controlled range of equations. In the opening review she had invited suggestions for the equation of a graph which "was going in the other direction, sloping the other way" to those of the form $y=x+c$ which were already displayed on the projected computer, and an unexpected response from one student was followed through with the whole class:

> It allows some of them to go off at a tangent and explore their own graphs. And a few of them were doing that with some quite interesting results. For example… when we looking at recapping linear graphs, [one boy] put in $x^2 = y^2$ and found out that it drew a line over $y = -x$, and then we discovered by putting up a blank sheet that it actually draws $y = -x$ and $y = x$. So we showed [by] simplifying the equation… [that] it would draw both of those graphs.

[13] Later in the lesson, the teacher encouraged students to use the software to explore beyond the types of equations specified in the textbook investigations:

> And that led some of them to explore quadratic graphs a little bit further. Some of them said "Well, it only makes a curve, a parabola, if you put in an $x^2 + 2$, but you have to keep the y as just a y, and not as a power." And then some of them tried $x^2 + y^2 = 25$ and got a graph of a circle. And there were quite a few different groups of people doing slightly different graphs, or investigating in a different way. But at the end, that was all brought together, and we discussed the different things that people had found out.

[14] In a similar vein, the teacher referred to a particular type of activity that she sometimes used at the start of computer-based lessons to give students scope to experiment with the operation of software, and so extend the features with which the class was familiar:

> Sometimes I do a five minute thing at the beginning of a lesson, where they've got to look at [the software] and explore, and… they can tell me anything exciting that they've found. And they quite like doing that. Some of them today found out how to shade inequalities, and we've been looking at inequalities in class, and so some of them just found that out on their own, which was great. I didn't have the chance to show the whole class today, but that will be something we will do.

Box 3: Lesson summary B1 – Investigating linear forms

[1] This one-hour lesson involved a Year 8 (Grade 7) class of average-attaining students. It took place in a computer suite with pairs of students seated together at a machine and a smartboard available to the teacher. There was no stylus for the smartboard, obliging the teacher to use finger touch and recalibrate the board. While this was adequate for giving graphing commands, it was less satisfactory for writing or drawing freehand.

[2] The resource system for the lesson consisted of a teacher-devised sequence of tasks involving use of graphing software. During the opening phase of the lesson, the teacher gave the class step-by-step instructions while students opened the graphing software on their machine, plotted a point, and entered an equation to graph a line.

[3] The lesson agenda was to review prerequisite material before introducing the new idea of linear graphs of the form $y = mx + c$ (through the examples $y = 2x + 3$, $y = 2x + 2$, $y = 2x + 4$) and then applying it in the tasks. As initially posed by the teacher, the main tasks were:

- Find the equation of a line which passes through the point (5, 2).
- Find the equation of a line which passes through the point (-1, 4).

[4] However, once a student pair had identified a suitable line they were pushed to find others "perhaps steeper or shallower or sloping in the other direction". This required them to explore altering the x coefficient in the equation:

> About half the class didn't vary the number of x's, they stuck with x and just varied the constant at the end. And they could find one and were like, "I've done it!" And I said, "Well, have you? Try another." And so the task initially started off with finding one line and developed into finding two, three, four.

[5] In terms of activity format, the lesson started with around 15 minutes of what was primarily whole-class, teacher-led exposition and questioning, but including instructing students in their first steps with the software. Two cycles then followed in each of which the teacher quickly posed a task to the whole class, launching around 15 minutes of student-pair work, with the teacher circulating to provide guidance and support, occasionally alerting the whole class to an issue. The lesson concluded with around 10 minutes of plenary review and elaboration of strategies and solutions, through whole-class, teacher-led exposition and questioning.

[6] The teacher singled out the way in which graphing software supported a different style of approach to the topic which allowed students to get a feel for linear forms within a single lesson:

> Just the different approach.… For this particular topic I wanted to really get them to feel what the values were doing rather than necessarily understanding straight away how the gradient value worked.… I just wanted them to get a feel of it and I wanted it to happen quite quickly. I only had one lesson and I wanted them to really get to grips with $y=mx+c$, which you can't really do… successfully in just one classroom lesson [without the software].

[7] More broadly, using technology enabled subsidiary procedures to be carried out rapidly and reliably:

> The thing the ICT offers maths [is] that you can do things extremely accurately and extremely quickly, things that sometimes take a laborious amount of time in a lesson. If you've got a class drawing axes and then finding co-ordinates and then plotting lines, it takes a lot longer than if they immediately see it on the screen and they can change it.

[8] Using the software made graphing more accessible to students, overcoming the presentational difficulties and physical impediments which many encountered when working with manual tools:

> I wanted them to get across the barrier of having to get a ruler and a pencil out. And there are lots of children in there who are not dyspraxic but certainly find organisation and presentation challenging. And today that barrier went completely and they could work as quickly, if not faster, than their peers, and also faster than they would expect themselves to work.… [The technology] does enable those students with particular difficulties to just overcome them immediately and just get on.

[9] Using graphing software also helped students to grasp the spatial patterning of the graphs and its link to their equations:

> We looked at seeing what the line $y = 2x + 1$ would look like.… And then looking at $y = 2x + 2$, most of them didn't know what was going to happen when I pressed 'enter'. But when they saw it they could then say "Well now I've seen that I think I know what $2x + 4$ would look like". And then [named student] chipped in with "What's $2x + 2.5$?" So they're getting an idea from what the c did, that it moved it up and down. I think that was successful and I think that was quite enlightening to a lot of the class.

[10] The teacher drew attention to the value of the software's colour coding in matching equations to graphs, but pointed out that students still needed to build a deeper understanding of this relationship:

> Omnigraph is nicely designed so that the equation of the line is the same colour as the line. I think that's a really useful part of this package, because if it wasn't you would easily get confused.… Between the graph and its equation, it's quite clear which is connected to which. Whether they understand why, why is that the equation of that line, or vice versa, I don't know.

[11] Equally, she reported particular difficulties which might arise and her strategies to pre-empt or retrieve them:

> Some of them were… having $y = x + -3$ and then $y = x – 3$, which are the same graph though the graph does change colour when they put the second one in, but it's not clear that that is the same graph. They just think that one has gone.
> Those little things. They try equations that won't work, or they try changing things in the equation. But the control was put on that today, because I thought about them doing $x =$ and said [that] I want[ed] everything to be $y =$. I put a little rule in there to stop or to restrict the amount of people that would vary that and do something different.

[12] Nevertheless, the teacher viewed play as an important aspect of using, and learning to use, software. Introducing the lesson, she told students that they needed "to have a chance to play with the software":

> Giving them the freedom to play with it a little bit… that's definitely what I would encourage. Almost with any computer package you need an element of play. If you just do structured activities and answer structured questions, you don't get to realise about what the package can do to make life a lot easier for you.

[13] The teacher had devised this lesson as "a target practice kind of thing", to be tackled through trial and improvement. However, she found herself having to persuade some students of its mathematical legitimacy:

> Some of them were very hesitant at trying their own thing… The whole idea of this activity is a trial and improvement activity so you are allowed to get things wrong. So I think once they'd got the idea that they were improving on their previous guess, that it didn't matter if it was wrong. Because many of them probably thought [they had to] find this exactly… to get it spot on, and that's not what the activity is designed to be.

Box 4: Lesson summary B2 – Investigating quadratic forms

[1] This one-hour lesson involved a Year 10 (Grade 9) class of high-attaining students. It took place in a computer suite with students seated mostly in pairs at a machine, but sometimes individually, and a smartboard available to the teacher. Many students were late for the lesson: awaiting their arrival, the teacher suggested that those students already present "have a little play" on the graphing software.

[2] The resource system for the lesson consisted of a teacher-devised worksheet calling for use of graphing software. The teacher reported that the class had already used the software several times so that she had no concerns about their capacity to graph an equation with it.

[3] The lesson agenda followed the sequence of worksheet tasks. The first task was to use the graphing software to explore the effect of altering particular coefficients of a quadratic equation on the shape and location of the resulting curve, with a set of three equations specified in relation to each coefficient (such as $y = x^2 + x + 1$, $y = x^2 + 3x + 1$, $y = x^2 - 4x + 1$). The second task was to find the equations of quadratic curves: first to pass through one specified point (3, 4), and then through a pair of specified points, (-1, 2) and (3, 5). Introducing this task to the class, the teacher pointed out its similarity to the "target practice" task they had met the previous year in work on straight line graphs.

[4] In terms of activity format and structure, the lesson started with around 10 minutes of whole-class, teacher-led exposition and questioning, reviewing prerequisite material and previewing the tasks. There then followed around 30 minutes of student work, individually or in pairs, with the teacher circulating to provide guidance and support. The lesson concluded with around 10 minutes of plenary review and elaboration of strategies and solutions, through whole-class, teacher-led exposition and questioning.

[5] The teacher had devised the worksheet in the light of her reservations about the one already available to her, which she felt brought a rather too structured and didactic form to the investigation:

> I wrote the worksheet, which was tailor-made to the lesson. I have got a [graphing software] sheet on quadratics and looked at it. [It] was very "Plot these graphs, what do you notice? Plot these graphs, where do they intersect?" and very much "Use the package to answer these questions". And I didn't want that feel to the lesson, I wanted the lesson to have this kind of discovery learning feel to it, so I decided to discard that and use a more open-ended worksheet. It was mainly to give them a chance to have somewhere to write down what they were thinking in words.

[6] Indeed, the teacher saw technology as providing particular support for a more independent style of discovery learning:

> With ICT you can do a lot of discovery aspects of learning. You can get them to sort of have their own control over the situation and I didn't tell them what equations to draw in. I gave them a few examples, but I didn't tell them the other one, they could do anything they like.

[7] Equally, she had designed the new worksheet with a concern that the new tasks should better interest, engage, and challenge students:

> I did try and make this activity slightly more interesting for them to do, and a more demanding activity. So I did, when it came to the worksheet, think carefully about how I was going to put the new one together so that it did what I wanted it to do without being laborious.

[8] One contribution of technology was in facilitating the rapid and reliable production of graphs:

> The idea was that [the technology] would make doing the activity an awful lot easier and quicker and more efficient… [so that] errors don't occur.

[9] The accessibility of this software to students assisted the ease with which the lesson could proceed:

> The package is very user-friendly… so you don't need to be a whiz on the computer to be able to master the skills, and be able to use the package to understand and interpret the mathematical things you are doing. So that was important and made the lesson very straightforward from that point of view.

[10] In terms of bringing out the target mathematical ideas, the straightforward way in which several graphs could be generated and compared assisted students in grasping the effect of altering particular coefficients, which was the focus of the first task:

> They actually got to grips with what those numbers are doing… what it looks like when you put a $2x^2$ as opposed to an x^2 and $3x^2$, and I think that's really important.

[11] However, while the effects of altering the parameters a and c were apprehensible to students, this was not so for b:

> I noticed on b a lot of them just wrote, "They all go through the point (0,1)" which was correct from what they had on their screen, but that wasn't quite what I wanted out of that particular activity. So I think there's a bit of tying up we need to do, and sharing of results and ideas.

[12] The teacher referred to a particular classroom episode as exemplifying the contribution of technology in helping students to take control of their own experimentation, and in supporting productive learning from such experimentation. It unfolded when one pair tried out an equation with a very small value for the coefficient of x^2:

> I think that certainly [named students]… started to use that aspect of the task, being able to have their own control and vary it. It was quite interesting when they put in that 0.00009[x^2], because it did bring out a really important point. … "Well, that's a straight line?"… "No it's not. I'm going to show you it's not." And they used the technology to help them do that. And then I said, "What about $0x^2 + x + 1$?"… And I could see the cogs turning, and he suddenly went, "$0x^2$. That'll just be $x + 1$. That would be a straight line!"… I think that was quite nice that they'd learned that if you have a very small amount of x^2, you do get a quadratic but you have to change the scales to see it, and it can initially look like a straight line. So I think that the technology helped there immensely because you couldn't do that on paper. It just wouldn't work.

[13] More broadly, the teacher wanted students to develop habits in which use of the graphing software was accompanied by more analytic processes of prediction and reflection, so as to make them more critical users:

> I wanted them to start to think about what a graph looks like before they actually press 'enter'. So when it comes to doing one in class again, what will that graph look like before they actually draw it. So they get some idea of what they are expecting. So if they get a point that's completely wrong, because they've interpreted something incorrectly when they are doing their table of results, they will know. "I know this graph should actually look like this. I know it should go through 1 on the y axis. I know it should be quite narrow. I know I'm expecting it to maybe shift to the left on the axes." And they've got some idea of what that graph is going to look like so then they can spot their own mistakes in the future.

5. A practitioner model of the contribution of graphing software to the teaching of algebraic forms

The themes from the earlier study provided a useful organising framework for synthesising the thinking reported by teachers in association with each lesson, making it possible to elaborate a practitioner model of the contribution of graphing software to the teaching of algebraic forms, grounded in the observation and interview data from this study.

Teacher accounts of all the lessons made reference to various aspects of the theme of *Effecting working processes and improving production,* suggesting, for example, that the software made it possible to produce graphs "extremely accurately and extremely quickly" [B1/7], making "doing the activity an awful lot easier and quicker and more efficient" [B2/8], so that – in terms of time economy – students could "move through everything at a much quicker pace" [A2/8], allowing a topic to be addressed in only a single lesson [A1/8; B1/6]. In the lower- and average-attaining classes, this also helped make tasks accessible to students who would have found "organisation and presentation challenging" [B1/8] and would "have really struggled" [A1/6], echoing aspects of *Overcoming pupil difficulties and building assurance.*

These factors also underpinned some aspects of the theme of *Enhancing the variety and appeal of classroom activity,* in terms of the use of graphing software making lessons less "laborious" [A1/6; B1/7; B2/7] and less dependent on pencil-and-paper work [A2/7; B1/8], and increasing the immediacy and interactivity of tasks [A1/5]. For the higher-attaining classes, the teachers also talked of the potential of using technology to make tasks more "challenging" [A2/5] or "demanding" [B2/7] in mathematical terms. With the lower-attaining class, technology was used to give students who "wanted to get the right thing in front of the class" [A1/7] the frisson of a very immediate and public test, by getting them to come out and check their proposal through using the software on the projected computer.

In relation to *Fostering pupil independence and peer exchange,* both teachers reported or were observed allotting short periods for playful exploration of the software by students, and for consequent sharing of discoveries [A1/15; A2/14; B1/12; B2/1]. Equally, for their higher-attaining classes, the teachers talked of using the technology to support "exploration" [A2/6] of more "open ended" tasks [B2/5], in which students "have their own control over the situation" [B2/6] and are "investigating, exploring, almost on their own" [A2/4]. Comparing her two lessons, Teacher A commented that she gave her higher-attaining class "more open-ended questions" to which they responded through "a text box on [their] graph to explain what differences [they had] seen", whilst her lower-attaining class were asked "much more particular questions" and "had a sheet to put very definite answers on… to focus them in". However, in the lower-attaining class [A1/14] as well as the higher-attaining classes [A2/12&13], when students came up with mathematical ideas going beyond the lesson agenda, she supported them in "go[ing] off at a tangent" [A2/12]. The contrast that Teacher B drew between the framing of tasks for her average- and higher-attaining classes was less strong: the former "had a very specific task", whereas the latter "had to do a slightly more open-ended task". She too was observed supporting students in going beyond the lesson agenda [B2/12] in a way which, as she pointed out, was only possible because of the availability of the graphing software.

Teachers' encouragement of informal exploration of the graphing software, and their assistance to students using it to engage in mathematical speculation and experimentation beyond the lesson agenda, also evidences how they saw this technology as a means of *Supporting processes of checking, trialling and refinement*. In both her lessons (suggesting that this had become part of her curriculum script), Teacher A posed the same speculative question about lines sloping in an opposite way, leading to a similar trialling episode being inserted into a conventional investigation [A1/13] and an introductory review [A2/12]. Likewise, the 'target practice' tasks in both of Teacher B's lessons (with the explicit linking of them [B2/3], indicating both a developing curriculum script, and an emerging activity format tailored to this type of topic) were conceived more broadly as examples of 'trial and improvement' [B1/13], dependent on feedback from the graphing software. With the younger class, this required the teacher to renegotiate norms with students who were hesitant about the legitimacy of trialling [B1/13]; with the older class, the socio-mathematical agenda had moved on to developing habits of prediction and reflection to scaffold trialling processes [B2/13].

Finally, in terms of the theme of *Focusing on overarching issues and accentuating important features,* the teachers talked of how use of graphing software helped students to "get to grips with" [B2/10], "get an idea of" [B1/9], or "see straight away" [A1/9] the effect of altering a coefficient in the equation on the properties of its graph. Likewise, the teachers highlighted particular software devices which facilitated apprehension of equation/graph matching [A1/10; B1/10], comparison of gradients [A1/10], and examination of limiting trends [A2/9]. Nevertheless, teacher management and guidance also played an important part in helping students to gain such insights. In relation to one of the 'target practice' tasks, for example, key actions of Teacher B included constraining the type of expression to be graphed [B1/11], drawing attention to the equivalence of expressions [B1/11], and pressing students to seek further equations so as to generate graphs which were "steeper or shallower or sloping in the other direction" [B1/4]. Likewise, in both her lessons, Teacher A reported actively checking, and if necessary developing, students' understanding of the relationship between the equation of a graph and the coordinates of points lying on it [A1/11; A2/10], and prompting students to attend to the key mathematical properties which investigations aimed to establish [A1/12; A2/11].

6. The significance of instrumental induction, task design, and teacher intervention

The discussion in the previous section has highlighted the crucial part played by teacher prestructuring of technology-based tasks and by teacher shaping of technology-and-task-mediated activity in realising the ideals of the practitioner model. This section develops these points to examine the ways in which teachers, and the wider sources that they drew on, contributed to the conduct of well functioning lessons.

Analysis of 'institutional' and 'instrumental' aspects of tool use (Artigue, 2002; Guin, Ruthven & Trouche, 2005; Ruthven, 2002) has developed in response to difficulties encountered with the educational use of sophisticated technologies designed for use by professional mathematicians. It provides a conceptual framework for analysing the process through which students (and indeed teachers) progressively appropriate a material artefact to create a mathematical instrument. Essentially, in the school context, this calls for development of an institutionalised order of tool-mediated activity, and the induction of users into this order. Graphing software, however, has been explicitly designed for educational use. The teachers described the packages they were using as "instinctive" [B] and "user-friendly" [A&B]. They identified several aspects of the user interface which made the software readily accessible and interpretable by students (contrasting the software favourably with graphic calculators in many of these respects): the clearly labelled scales [A&B] and the gridlines in the background to assist comparison of gradients [A]; the 'hand' tool for dragging the image to view sections of the graph outside the original display [A]; the colour coding which associated particular equations with their graphs when several were displayed simultaneously [A&B]; the acceptability of expressions defined in the form $x=$ as well as $y=$, and defined implicitly as well as explicitly [A]. Nevertheless, however much these graphing packages had been "designed to do things easily" [B], the teachers still played an important role in inducting and supporting students in use of the software for mathematical purposes. It was on this foundation that classroom realisation of the teachers' conceptions of successful technology use depended.

Both teachers followed a dual approach to establishing a collective repertoire of computer graphing techniques. Prior to undertaking a classroom task involving graphing, unless the teachers had confidence that the core techniques required were already familiar to the class [B2/2], they introduced [A1/2; B1/2] or reviewed [A2/2] them. More serendipitously, they also allotted short periods to playful exploration of the software by students, and to subsequent sharing of new possibilities [A1/15; A2/14; B1/12; B2/1]. Supporting and developing use of the software was also an important dimension of teacher interaction with students while they were working on tasks. In the observed lessons, teachers guided basic operation of the software, prompted strategic action with it, and supported mathematical interpretation of its results. Teacher actions included: explaining how to enlarge a target point to make it more visible [B1], and how to enter x^2 in the equation editor [B2]; helping students to understand why the software had produced a horizontal line rather than the expected sloping one (as a result of entering y=5+4 rather than y=5x+4) [B1], or a straight line rather than the expected curve (as a result of entering y=x+2^2 rather than y=$(x+2)^2$) [B2]; prompting students to drag the displayed image to expose more of a particular graph [A1], or to pursue the limiting trend of a graph [A2]; and prompting students to zoom out on the displayed image of $0.00000009x^2+x+1$ to test whether it was a straight line, then introducing the comparison with $0x^2+x+1$ [B2].

Likewise, the structuring of lesson tasks through prepared materials and teacher intervention was crucial in realising many of the benefits attributed to using the graphing software. The mathematical content of the prepared tasks used by teachers corresponded closely to examples suggested in official guidance, as illustrated by the extracts in Box 5. The prepared tasks used in both lessons by Teacher A followed the inductive format exemplified by official examples *1* and *2,* as did the first prepared task used in the second lesson by Teacher B. Teacher B's 'target practice' tasks can be viewed as a reframing of the format exemplified by official example *3*, capitalising on the interactivity of the technology. Her reason for adopting this task format was to introduce a more exploratory style, breaking away from what she saw as the overly didactic style of the investigation genre [B2/5]. In the classroom, however, her intervention proved necessary to reframe the task in suitably didactic terms [B1/4]. This signals the crucial part that didactical structuring through task design and/or teacher intervention plays in *Focusing on overarching issues and accentuating important features*.

Box 5: Exemplary lesson tasks from the official teaching guidance for Years 8 and 9

Plot the graphs of simple linear functions, in the form $y = mx + c$, on paper or using ICT, and consider their features.

1) Plot and interpret graphs such as:
   $y = 2x$ $\quad$ $y = 2x + 1$ $\quad$ $y = 2x + 4$ $\quad$ $y = 2x - 2$ $\quad$ $y = 2x - 5$ $\quad$ [p. 165]

Use a graphical calculator to investigate the family of straight lines $y = mx + c$.

2) Draw the graphs of:
   $y = x + 1$ $\quad$ $y = x + 2$ $\quad$ $y = x + 3$
   $y = x - 1$ $\quad$ $y = x - 2$ $\quad$ $y = x - 3$
   Describe what the value of m represents.
   Describe what the value of c represents.
3) Use a graphical calculator and knowledge of the graph of $y = mx + c$ to explore drawing lines through:
   (0, 5) $\quad$ ($^{-}7$, $^{-}7$) $\quad$ (2, 6)
   ($^{-}7$, 0) and (0, 7) $\quad$ ($^{-}3$, 0) and (0, 6) $\quad$ (0, $^{-}8$) and (8, 0) $\quad$ [p. 167]

Use a graphical calculator to investigate graphs of functions of the form $y = ax^2 + bx + c$, for different values of *a, b* and *c.*

4) Investigate families of curves such as:
   $y = ax^2$ $\quad$ $y = (x + b)^2$ $\quad$ $y = x^2 + c$ $\quad$ $y = (x + b)^2 + c$ $\quad$ [p. 171]

Essentially, whatever task format was adopted, the learning goal of these lessons was to induct students into an accepted mathematical organisation of the multimodal systems constituted by equations of the types $y = mx + c$ or $y = ax^2 + bx + c$ and their graphs. Achieving such an organisation depends on managing the double semiotic of the system through coordinating algebraic and geometric registers, while also managing its multi-dimensionality through isolating phenomena and controlling variables. The official example *4* (in Box 5) illustrates this (as do the example types and sets in Boxes 1-4). Each of the first three subtasks in this example isolates a geometric phenomenon and controls algebraic form accordingly, playing on a single parameter; only with the fourth subtask does some integration of these phenomena commence. Nevertheless, a breakdown in such management emerged in one quadratic investigation where students failed to formulate the intended property of the coefficient *b* from the family of forms selected to exemplify it [B2/11]. This example apart, however, the investigation tasks (and indeed the lesson expositions) employed by teachers were largely successful – through sequencing and patterning example types and sets – in providing a logical decomposition of the multimodal mathematical system under consideration. It is this didactical organisation of the topic which underpinned the use of graphing software to help students grasp the effect of altering a coefficient in the equation on the appearance of its graph. In the absence of such structuring, however, the 'target practice' tasks called for much higher levels of teacher mediation.

This puts in perspective, on one side, Teacher A's suggestion that students were "investigating, exploring, almost on their own", and, on the other, Teacher B's (partial) renunciation of the "'Do this, what do you notice? Do this, what do you notice?" style of the investigation genre. We have seen that the teachers were certainly aware of structuring their introductory expositions and student investigations to differing degrees for their two classes. But whatever the class, realising the benefits of using the graphing software to enhance the generation and comparison of multiple examples was dependent to a large extent on the intellectual organisation provided by other prepared resources and further teacher contributions. In the investigation tasks and introductory expositions such organisation was introduced through carefully structured example sets. But because the 'target practice' tasks incorporated no comparable structuring, given students' limited experience and knowledge of the mathematical topics in question, the organising function then fell predominantly on teacher prompting and questioning. This emphasises that graphing software and lesson tasks form a resource system, in which the technology's contribution to supporting learning is powerfully conditioned by task structuring

7. The adaptation of teaching practices and the development of craft knowledge

Earlier sections have highlighted the crucial part played by teacher structuring and shaping of technology-mediated activity in realising the ideals of the practitioner model. These sections have evidenced the adaptation of teaching practices and development of craft knowledge associated with teachers appropriating graphing software as an instrument for teaching and learning mathematics.

In terms of *working environment*, many of the aspects observed were not specific to graphing software. In three of the four lessons [A1; A2; B1], for example, technical difficulties arose which required some modification of normal working procedures. Likewise, in the changed working environment of the computer suite, teachers had to modify *classroom routines*, notably those concerned with managing the start of lessons, to include getting students seated appropriately, and their computer workstations and resources opened for use. Equally, adaptation was required to routines for securing the attention of students during periods of independent work, so as to efficiently make important points to the class as a whole [B1]:

> I did a little [whole-class intervention during] the main activity… because if you didn't do that you'd have repetitive questions. Which is something I'd do in a lesson anyway, so it's just translating that to computers, and getting them to turn their monitors off and face the board… That's something I'd be quite keen to get them into the habit of doing… Tearing them away from their computers… I find that very difficult because you know they're not listening to you and they are missing out on something.

Developing a functional *resource system* incorporating the use of graphing software required teachers to extend their practice in several ways. As reported earlier, they developed strategies both to familiarise students with (and later to review) core techniques for using the software, and to allow students to explore (and then to share their discoveries of) a wider range of technical possibilities. Complementarily, the teachers had devised or appropriated suitable tasks and supporting textual materials to underpin classroom activity that employed computer graphing to investigate the topic of algebraic forms. They were also developing a repertoire of strategies to support students in tackling these tasks, concerned not just with guiding software operation, but with prompting strategic action, and supporting mathematical interpretation.

In terms of *activity structures*, teachers suggested that use of technology made investigative lessons more viable. Equally, it seems that the availability of projection facilities permitted all the investigative lessons observed in this study to be organised within an activity structure in which episodes of individual or paired student activity at workstations were interleaved with whole-class activity, concluding with plenary review. Moreover, in the practice of Teacher B, the emergent type of 'target practice' task was associated with a rather different *activity format* for individual or paired student work, capitalising on the interactivity of the software to centre investigative activity around a process of trial and improvement. Notable also was the similar way in which both teachers had adapted the whole-class exposition and questioning format to exploit the opportunity to use the software to provide immediate feedback on student predictions, for example by students 'taking the stage' to use the projected computer to test their suggestions.

These preceding elements of adaptation are all interwoven in the development of teachers' *curriculum scripts* for the topic of algebraic forms, as evidenced in the *lesson agendas* they formulated, and in the detail of their classroom action (including interaction) during the observed lessons. On the basis of explicit comment by teachers (such as Teacher B referring her older class back to their previous encounter with the 'target practice' genre) or recurrent patterns of teacher action (such as Teacher A posing to both her classes the same speculative question leading to similar trialling activity) some of these examples clearly represent mature developments in teachers' curriculum scripts for the topic. Other examples provide more evidence of teachers extending their repertoire of approaches to supporting students and (re)directing them towards desired states, intended responses and resultant learning. This included teachers' extension of their capacity for reactive teaching in response to new types of student initiative made possible by the graphing software (such as Teacher A responding to students' observations of the overlap of graphs [A1/14; A2/12] by explaining these properties through symbolic reduction of the more complex equation; and Teacher B guiding students' exploration of a 'barely' quadratic expression through zooming out on its graph and comparing it to the 'null-ly' quadratic expression [B2/12]).

Finally, change in the *time economy* is evident in teachers' comments on the contribution of graphing software to *Effecting working processes and improving production.* Equally, teachers reported that use of the software improved rate of learning return from classroom time by virtue of other contributions identified in the practitioner model, notably *Overcoming pupil difficulties and building assurance* and *Focusing on overarching issues and accentuating important features.* On the other hand, teachers had to manage the development of a double instrumentation of graphing both by hand and by machine. Doing this efficiently was assisted by development of a coherent resource system as noted earlier, and by the availability of classroom projection permitting collective instrumented activity involving the class as a whole.

8. Conclusion

This study of teacher thinking and classroom practice can be read at two levels: at a more specific level it seeks to provide a holistic understanding of how English secondary-school mathematics teachers use graphing software to teach about algebraic forms; at a more generic level, it aims to provide an example that illustrates the potential of an evolving framework for understanding technology use in school mathematics teaching.

Viewed from a strictly mathematical perspective, the archetypical classroom usage of graphing software that has been identified and examined in this study focuses on those same ideas identified in Fey's (1989) discussion of pioneering work some twenty years ago: namely relations between symbolic expressions and coordinate graphs, notably the connection between particular coefficients or parameters in an expression and particular features of the corresponding graph. Likewise the same types of mathematical task and attendant reasoning are prominent: namely induction of relationships between expression and graph through identification of pattern, and matching of expression to graph through trial and improvement. What this study shows, however, is that these features are simply the mathematical face of a larger pedagogical adaptation. Here indeed, both in official guidance and in classroom practice, graphing software was treated essentially as a pedagogical aid (replicating the predominant trend evidenced in the earlier work reviewed).

Teachers were particularly drawn to use graphing technology to support classroom activity that they variously described as involving investigation, exploration or discovery (as reported also in earlier studies by Farrell, 1996; Simmt, 1997). More specifically, in terms of the general themes of the practitioner model developed in our earlier work, teachers saw graphing software as contributing to:

- *Effecting working processes and improving production* through making it easier to produce graphs accurately and rapidly, so increasing the efficiency and pace with which related topics can be covered;
- *Overcoming pupil difficulties and building assurance,* through making graphing tasks more accessible to students who have difficulties with organisation and presentation;
- *Supporting processes of checking, trialling and refinement,* through enabling lesson tasks based on trial and improvement, and supporting mathematical speculation and experimentation within and beyond the lesson agenda;
- *Focusing on overarching issues and accentuating important features,* through helping to bring out the effects of altering particular coefficients or parameters in an equation on the properties of its graph, and through facilitating comparison of gradients and examination of limiting trends;
- *Enhancing the variety and appeal of classroom activity*, through reducing 'laborious' written work, increasing the immediacy and interactivity of classroom tasks, and helping to create new forms of playful challenge within lessons;
- *Fostering pupil independence and peer exchange*, through providing support for exploration by students and consequent sharing of discoveries, including software techniques and mathematical ideas within and beyond the lesson agenda.

At the same time, this study has highlighted the crucial part played by teacher structuring and shaping of technology-and-task-mediated student activity (as also shown in earlier studies by Doerr & Zangor, 2000; Farrell, 1996; Godwin & Sutherland, 2004) in realising the ideals of the practitioner model. Although teachers consider graphing software very accessible, successful classroom use still depends on their inducting students into using computer graphing for mathematical purposes, providing suitably prestructured lesson tasks, prompting strategic use of the software by students, and supporting mathematical interpretation of the results. Equally, (as noted also by Godwin & Sutherland, 2004), the example of teaching about algebraic forms illustrates how, by providing prestructuring lesson tasks and intervening to shape student work on them, teacher contributions play a fundamental part in managing the underlying semiotic system to make mathematical relations apprehensible to students, through coordinating algebraic and geometric registers, isolating phenomena and controlling variables.

Finally, this study has illustrated ways in which teachers, in the course of appropriating graphing software, adapt their classroom practice and develop their craft knowledge. Specifically, it has highlighted how teachers:

- establish a coherent resource system incorporating software-mediated lesson tasks aligned with teaching goals, and supported by a common repertoire of suitable graphing techniques;
- adapt formats for classroom activity to capitalise on the interactivity of the software;
- extend curriculum scripts to encompass these features, and to provide for proactive structuring and responsive shaping of student activity on software-mediated lesson tasks;
- rework lesson agendas, both to include induction to computer graphing and to take advantage of the resulting time economy.

Acknowledgements

Particular thanks are due to the teacher colleagues featured in the case studies; to Theresa Daly our project secretary; and to the UK Economic and Social Research Council which funded the *Eliciting Situated Expertise in ICT-integrated Mathematics and Science Teaching* project (R000239823). An earlier report of this study was presented at the November 2008 day-conference of the British Society for Research in Learning Mathematics: we are grateful to the BSRLM audience as well as to the ESM editor and reviewers for their helpful comments and suggestions.

References

Amarel, M. (1983). Classrooms and computers as instructional settings. *Theory into Practice* 22, 260-266.

Artigue, M. (2002). Learning mathematics in a CAS environment: The genesis of a reflection about instrumentation and the dialectics between technical and conceptual work. *International Journal of Computers for Mathematical Learning,* 7(3), 245-274.

Assude. T. (2005). Time management in the work economy of a class. A case study: Integration of Cabri in primary school mathematics teaching. *Educational Studies in Mathematics,* 59(1-3), 183-203.

Brown, S., & McIntyre, D. (1993). *Making Sense of Teaching.* Buckingham: Open University Press.

Burns, R. B., & Anderson, L. W. (1987). The activity structure of lesson segments. *Curriculum Inquiry,* 17(1), 31-53.

Burns, R. B., & Lash, A. A. (1986). A comparison of activity structures during basic skills and problem-solving instruction in seventh-grade mathematics. *American Educational Research Journal,* 23(3), 393-414.

Caliskan-Dedeoglu, N. (2006). Usages de la géométrie dynamique par des enseignants de collège. Des potentialités à la mise en oeuvre: quelles motivations, quelles pratiques. Unpublished doctoral thesis, University of Paris 7.

Department for Education and Employment [DfEE] (2001). *Key Stage 3 National Strategy: Framework for teaching mathematics.* London: DfEE.

Department for Education and Skills [DfES] (2003). *Integrating ICT into Mathematics at Key Stage 3.* London: DfES.

Doerr, H. & Zangor, R. (2000). Creating meaning for and with the graphing calculator. *Educational Studies in Mathematics,* 41(2), 143-163.

Farrell, A. M. (1996). Roles and behaviors in technology-integrated precalculus classrooms. *Journal of Mathematical Behavior,* 15(1), 33-53.

Fey, J. (1989). Technology and mathematics education: A survey of recent developments and important problems. *Educational Studies in Mathematics,* 20(3), 237-272.

Godwin, S. & Sutherland, R. (2004). Whole-class technology for learning mathematics: the case of functions and graphs. *Education, Communication & Information,* 4(1), 131-152.

Guin, D., Ruthven, K., & Trouche, L. (Eds.) (2005). *The didactical challenge of symbolic calculators: turning a computational device into a mathematical instrument.* New York: Springer.

Jenson, J. & Rose, C B. (2006). Finding space for technology: Pedagogical observations on the organization of computers in school environments. *Canadian Journal of Learning and Technology,* 32(1). Accessed at http://www.cjlt.ca/content/vol32.1/jenson.html

Lagrange, J.-B. & Caliskan-Dedeoglu, N. (in press). Usages de la technologie dans des conditions ordinaires: Le cas de la géométrie dynamique au collège: Potentialités, attentes, pratiques. *Recherches en Didactique des Mathématiques*.

Leinhardt, G. (1988). Situated knowledge and expertise in teaching. In J. Calderhead (Ed.), *Teachers' Professional Learning* (pp. 146-168). London: Falmer.

Leinhardt, G., Putnam T., Stein, M.K., & Baxter, J. (1991). Where subject knowledge matters. *Advances in Research in Teaching,* 2, 87-113.

Leinhardt, G., Weidman, C., & Hammond, K. M. (1987). Introduction and integration of classroom routines by expert teachers. *Curriculum Inquiry,* 17(2), 135-176.

Monaghan, J. (2004). Teachers' activities in technology-based mathematics lessons. *International Journal of Computers for Mathematical Learning,* 9(3), 327-357.

Office for Standards in Education [OfStEd] (2004). *ICT in schools – the impact of government initiatives: Secondary mathematics.* London: OfStEd.

Ruthven, K. (2002). Instrumenting mathematical activity: Reflections on key studies of the educational use of computer algebra systems. *International Journal of Computers for Mathematical Learning,* 7(3), 275-291.

Ruthven, K. (2007). Teachers, technologies and the structures of schooling. In D. Pitta-Pantazi & G. Philippou (Eds.), *Proceedings of the Fifth Congress of the European Society for Research in Mathematics Education*, pp. 52-67.

Ruthven, K. (in press). Towards a naturalistic conceptualisation of technology integration in classroom practice: The example of school mathematics, *Education & Didactique*.

Ruthven, K., & Hennessy, S. (2002). A practitioner model of the use of computer-based tools and resources to support mathematics teaching and learning. *Educational Studies in Mathematics,* 49(1), 47-88.

Ruthven, K., & Hennessy, S. (2003). Successful ICT use in secondary mathematics – a teacher perspective. *Micromath,* 19(2), 20-24.

Ruthven, K., Hennessy, S., & Brindley, S. (2004). Teacher representations of the successful use of computer-based tools and resources in secondary-school English, Mathematics and Science. *Teaching and Teacher Education,* 20(3), 259-275.

Ruthven, K., Hennessy, S., & Deaney, R. (2005). Incorporating dynamic geometry systems into secondary mathematics education: Didactical perspectives and practices of teachers. In D. Wright (Ed.), *Moving on with Dynamic Geometry* (pp. 138-158). Association of Teachers of Mathematics: Derby.

Ruthven, K., Hennessy, S., & Deaney, R. (2008). Constructions of dynamic geometry: a study of the interpretative flexibility of educational software in classroom practice. *Computers and Education,* 51(1), 297-317.

Simmt, E. (1997). Graphing calculators in high school mathematics. *Journal of Computers in Mathematics and Science Teaching,* 16(2/3), 269-289.

Strauss, A., & Corbin, J. (1994). Grounded theory methodology: an overview. In N. K. Denzin & Y. S. Lincoln (Eds.), *Handbook of Qualitative Research*. London: Sage.

Wilson, S. M., Shulman, L. S., & Richert, A. E. (1987). '150 different ways' of knowing: Representations of knowledge in teaching. In J. Calderhead (Ed.), *Exploring Teachers' Thinking* (pp. 104-124). London: Cassell.