AN INVESTIGATIVE LESSON WITH DYNAMIC GEOMETRY: A CASE STUDY OF KEY STRUCTURING FEATURES OF TECHNOLOGY INTEGRATION IN CLASSROOM PRACTICE

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The research literatures concerning the classroom practice of mathematics teaching and technology integration in school mathematics point to key structuring features – working environment, resource system, activity format, curriculum script, time economy – that shape patterns of technology integration into classroom practice and require teachers to develop their craft knowledge accordingly. This conceptual framework is applied to an investigative lesson incorporating dynamic geometry use, employing evidence from classroom observation and teacher interview. This illuminates the many aspects of professional adaptation and development on which successful technology integration into classroom practice depends.

INTRODUCTION TO THE STUDY

From synthesis of relevant research literatures, I argued at CERME-5 that successful integration of computer-based tools and resources into school mathematics depends on coordinating working environment, resource system, activity format and curriculum script to underpin classroom practice which is viable within the time economy (Ruthven, 2007). This paper will illustrate –and test– that conceptual framework by using it to analyse the practitioner thinking and professional learning surrounding a lesson incorporating the use of dynamic geometry.

The lesson was one of four cases investigated in a study of classroom practice incorporating dynamic geometry use (Ruthven, Hennessy & Deaney, 2008). In the original study, this specific case was followed up because the teacher concerned talked lucidly about his experience of teaching such a lesson for the first time, and displayed particular awareness of the potential of dynamic geometry for developing visuo-spatial and linguistic aspects of students' geometrical thinking.

This case has been chosen for further analysis because the teacher was unusually expansive in all his interviews, illuminating a range of aspects of practitioner thinking and professional learning. While an exhaustive case analysis in terms of the conceptual framework would require data to be collected with its use specifically in mind, the richness of the evidence from this case provides a convenient interim means of exploring its application to a concrete example.

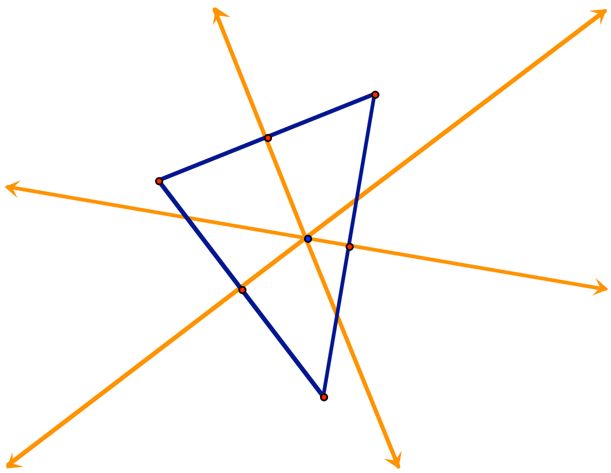
ORIENTATION TO THE LESSON

As the teacher explained when nominating the lesson, it had recently been developed in response to improved technology provision in the mathematics department prompting him to "*to explore some geometry*":

> So we'd done some very rough work on constructions with compasses and bisecting triangles and then I extended that to Geometer's Sketchpad… on the interactive whiteboard using it in front of the class.

He reported that the lesson (with a class in the early stages of secondary education) had started with him constructing a triangle, and then the perpendicular bisectors of its edges. The focus of the investigation which ensued had been on the idea that this construction might identify the 'centre' of a triangle:

> And we drew a triangle and bisected the sides of a triangle and they noted that they all met at a point. And then I said: "Well let's have a look, is that the centre of a triangle?" And we moved it around and it wasn't the centre of the triangle, sometimes it was inside the triangle and sometimes outside.



According to the teacher, one particularly successful aspect of the lesson had been the extent to which students actively participated in the investigation:

> *And they were all exploring; sometimes they were coming up and actually sort of playing with the board themselves… I was really pleased because lots of people were taking part and people wanted to come and have a go at the constructions.*

Indeed, because of the interest and engagement shown by students, the teacher had decided to extend the lesson into a second session, held in a computer room to allow students to work individually at a computer:

> *And it was clear they all wanted to have a go so we went into the computer room for the next lesson so they could just continue it individually on a computer… I was expecting them all to arrive in the computer room and say: "How do you do this? What do I have to do again?"… But virtually everyone… could get just straight down and do it. I was really surprised. And the constructions, remembering all the constructions as well.*

For the teacher, then, this recall by students of ideas from the earlier session was another aspect of the lesson's success. In terms of the specific contribution of dynamic geometry to this success, the teacher noted how the software supported exploration of different cases, and overcome the practical difficulties which students encountered in using classical tools to attempt such an investigation by hand:

> *You can move it around and see that it's always the case and not just that one off example. But I also think they get bogged down with the technicalities of drawing the things and getting their compasses right, and [with] their pencils broken.*

But the teacher saw the contribution of the software as going beyond ease and accuracy; using it required properties to be formulated precisely in geometrical terms:

> *And it's the precision of realising that the compass construction… is about the definition of what the perpendicular bisector is… And Geometer's Sketchpad forces you to use the geometry and know the actual properties that you can explore.*

These, then, were the terms in which the original lesson was nominated as an example of successful practice. This nomination was followed up by studying a later lesson along similar lines through classroom observations and teacher interviews. The observed lesson was conducted over two 45-minute sessions on consecutive days with a Year 7 class of students (aged 11-12) in their first year of secondary education.

WORKING ENVIRONMENT

The use of ICT in teaching often involves changes in the working environment of lessons: change of room location and physical layout, change in class organisation and classroom procedures.

Each session of the observed lesson started in the normal classroom and then moved to a nearby computer suite, a modification of the pattern originally reported. This movement between rooms allowed the teacher to follow a particular activity cycle common to each session, shifting working environment to match changing activity format. The classroom was equipped with a single computer linked to a ceiling-mounted projector directed towards a whiteboard at the front: this supported use of computer-based resources within whole-class activity formats. However, only in the computer suite was it possible for students to work individually at a machine.

Even though the suite was also equipped with a projectable computer, starting sessions in the teacher's own classroom was expedient for several reasons. Doing so avoided disruption to the established routines underpinning the smooth launch of lessons. Moreover, the classroom provided an environment more conducive to sustaining effective communication during whole-class activity and to maintaining the attention of students. Whereas in the computer suite each student was seated behind a sizeable monitor perched on a desktop computer unit, so blocking lines of sight and placing diversion at students' fingertips, in the classroom the teacher could introduce the lesson *"without the distraction of computers in front of each of them"*.

It was only recently that the classroom had been refurbished and equipped, and a neighbouring computer suite established for the exclusive use of the mathematics department. The teacher contrasted this new arrangement favourably in terms of the easier and more regular access to technology that it afforded, and the consequent increase in the fluency of students' use:

> *Before… you'd book a computer suite, you'd go in and then… you'[d] just not get anywhere, because the whole lesson's been sorting out logging on, sorting out how to use [the software]… And [now] having the access to it so easily and readily just makes a huge difference.*

New routines were being introduced to students for opening a workstation, including logging on to the school network, using shortcuts to access resources, and maximising the document window. Likewise, routines were being developed for closing sessions in the computer suite. Towards the end of each session, the teacher prompted students to plan to save their files and print out their work, advising them that he'd *"rather have a small amount that you understand well than loads and loads of pages printed out that you haven't even read"*. He asked students to avoid rushing to print their work at the end of the lesson, and explained how they could adjust their output to try to fit it onto a single page; he reminded them to give their file a name that indicated its contents, and to put their name on their document to make it easy to identify amongst all the output from the single shared printer.

RESOURCE SYSTEM

New technologies have broadened the types of resource available to support school mathematics. Nevertheless, there is a great difference between a collection of resources and a coherent system.

The department maintained its own schemes of work under continuous development, with teachers encouraged to explore new possibilities and report to colleagues. This meant that they were accustomed to integrating material from different sources into a common scheme. However, so wide was the range of computer-based resources currently being trialled that our informant (who was head of department) expressed concern about incorporating them effectively into departmental schemes:

> *At the moment we're just dabbling in [a variety of technologies and resources] when people feel like it, but we're moving towards integrating [them] into schemes of work now… I'm slightly worried that we've got so much… It's getting everybody familiar with it all.*

In terms of coordinating use of old and new technologies, work with dynamic geometry was seen as complementing established work on construction by hand, by strengthening attention to the related geometric properties:

> *I thought of Geometer's Sketchpad [because] I wanted to balance the being able to actually draw [a figure] with pencil and compasses and straight edges, with also seeing the geometrical facts about it as well. And sometimes [students] don't draw it accurately enough to get things like that all the [perpendicular bisectors] meet at the orthocentre[1] of the circle.*

The accuracy, speed and manipulative ease of dynamic geometry facilitated geometrical investigations which were difficult to undertake by hand:

> *[It] takes hours and hours if you try and do that by pencil and paper… So just that power of Geometer's Sketchpad to move the triangle around and try different triangles within seconds was fantastic. Ideal for this sort of exploration.*

Nevertheless, the teacher felt that old and new tools lacked congruence, because certain manual techniques appeared to lack computer counterparts. Accordingly, old and new were seen as involving different methods and having distinct functions:

> *When you do compasses, you use circles and arcs, and you keep your compasses the same. And I say to them: "Never move your compasses once you've started drawing."… Well Geometer's Sketchpad doesn't use that notion at all… So it's a different method.… I don't think there's a great deal of connection. I don't think it's a way of teaching constructions, it's a way of exploring the geometry.*

Equally, some features of computer tools were not wholly welcome: students could be deflected from the mathematical focus of a task by overconcern with presentation. During this lesson the teacher had tried out a new technique for managing this, by briefly projecting a prepared example to show students the kind of document that they were expected to produce, and illustrating appropriate use of colour coding:

> *They spend about three quarters of the lesson making the font look nice and making it all look pretty [but] getting away from the maths.… I've never tried it before, but that showing at the end roughly what I wanted them to have would help. Because it showed that I did want them to think about the presentation, I did want them to slightly adjust the font and change the colours a little bit, to emphasise the maths, not to make it just look pretty.*

Here we see the development of sociomathematical norms for using new technologies, and classroom strategies for establishing and maintaining these norms. Likewise, the way in which dynamic geometry required clear instructions to be given in precise mathematical terms was conveyed as being its key characteristic:

> *I always introduce Geometer's Sketchpad by saying: "It's very specific, you've got to tell it. It's not just drawing, it's drawing using mathematical rules."… They're quite happy with that notion of… the computer only following certain clear instructions.*

ACTIVITY FORMAT

Classroom activity is organised around formats for action and interaction which frame the contributions of teacher and students to particular lesson segments (Burns & Anderson, 1987). The crafting of lessons around familiar activity formats and their supporting classroom routines helps to make them flow smoothly in a focused, predictable and fluid way (Leinhardt, Weidman & Hammond, 1987). This leads to the creation of prototypical activity structures or cycles for particular styles of lesson.

Each session of the observed lesson followed a similar activity cycle, starting with teacher-led activity in the normal classroom, followed by student activity at individual computers in the nearby computer suite, and with change of rooms during sessions serving to match working environment to activity format. Indeed, when the teacher had first nominated this lesson, he had remarked on how it combined a range of classroom activity formats to create a promising lesson structure:

There was a bit of whole class, a bit of individual work and some exploration, so it's a model that I'd like to pursue because it was the first time I'd done something that involved quite all those different aspects.

In discussing the observed lesson, however, the teacher highlighted one aspect of the model which had not functioned as well as he would have liked: the fostering of discussion during individual student work. He identified a need for further consideration of the balance between opportunities for individual exploration and productive discussion, through exploring having students work in pairs:

There was not as much discussion as I would have liked. I'm not sure really how combine working with computers with discussing. You can put two or three [students] on a computer, which is what you might have done in the days when we didn't have enough computers, but that takes away the opportunity for everybody to explore things for themselves. Perhaps in other lessons… as I develop the use of the computer room I might decide… [to] work in pairs. That's something I'll have to explore.

At the same time, the teacher noted a number of ways in which the computer environment helped to support his own interactions with students within an activity format of individual working. Such opportunities arose from helping students to identify and resolve bugs in their dynamic geometry constructions:

[Named student] had a mid point of one line selected and the line of another, so he had a perpendicular line to another, and he didn't actually notice which is worrying… And that's what I was trying to do when I was going round to individuals. They were saying: "Oh, something's wrong." So I was: "Which line is perpendicular to that one?"

Equally, the teacher was developing ideas about the pedagogical affordances of text-boxes, realising that they created conditions under which students might be more willing to consider revising their written comments:

And also the fact that they had a text box… and they could change it and edit it. They could actually then think about what they were writing, how they describe, I could have those discussions. With handwritten, if someone writes a whole sentence next to a neat diagram, and you say: "Well actually, what about that word? Can you add this in?" You've just ruined their work. But with technology you can just change it, highlight it and add on an extra bit, and they don't mind.

This was helping him to achieve his goal of developing students' capacity to express themselves clearly in geometrical terms:

I was focusing on getting them to write a rule clearly. I mean there were a lot writing "They all meet" or even, someone said "They all have a centre."… So we were trying to discuss what "all" meant, and a girl at the back had "The perpendicular bisectors meet", but I think she'd heard me say that to someone else, and changed it herself. "Meet at a point": having that sort of sentence there.

CURRICULUM SCRIPT

In planning and conducting lessons on a topic, teachers draw on a loosely ordered model of relevant goals and actions that guides their teaching. This forms what has been termed a ‘curriculum script’ – where ‘script’ is used in the psychological sense of a form of event-structured cognitive organisation, which includes variant expectancies of a situation and alternative courses of action (Leinhardt, Putnam, Stein & Baxter, 1991). This script includes tasks to be undertaken, representations to be employed, activity formats to be used, and student difficulties to be anticipated.

The observed lesson followed on from earlier ones in which the class had undertaken simple constructions with classical tools: in particular, using compasses to construct the perpendicular bisector of a line segment. Further evidence that the teacher’s script for this topic originated prior to the availability of dynamic geometry was his reference to the practical difficulties which students encountered in working by hand to accurately construct the perpendicular bisectors of a triangle. His evolving script now included knowledge of how software operation might likewise derail students’ attempts to construct perpendicular bisectors, and of how such difficulties might be turned to advantage in reinforcing the mathematical focus of the task:

> *Understanding the idea of perpendicular bisector… you select the line and the [mid]point… There’s a few people that missed that and drew random lines… And I think they just misunderstood, because one of the awkward things about it is the selection tool. If you select on something and then you select another thing, it adds to the selection, which is quite unusual for any Windows package… So you have to click away and de-select things, and that caused a bit of confusion, even though I had told them a lot. But… quite a few discussions I had with them emphasised which line is perpendicular to that edge… So sometimes the mistakes actually helped.*

Equally, the teacher’s curriculum script anticipated that students might not appreciate the geometrical significance of the concurrence of perpendicular bisectors, and incorporated strategies for addressing this:

> *They didn’t spot that [the perpendicular bisectors] all met at a point as easily… I don’t think anybody got that without some sort of prompting. It’s not that they didn’t notice it, but they didn’t see it as a significant thing to look for… even though there were a few hints in the worksheet that that’s what they were supposed to be looking at, because I thought that they might not spot it. So I was quite surprised… that they didn’t seem to think that three lines all meeting at a point was particularly exceptional circumstances. I tried to get them to see that… three random lines, what was the chance of them all meeting at a point.*

The line of argument alluded to here was one already applicable in a pencil and paper environment. Later in the interview, however, the teacher made reference to another strategy which brought the distinctive affordances of dragging the dynamic figure to bear on this issue:

When I talked about meeting at a point, they were able to move it around, and I think there's more potential to do that on the screen.

Likewise, his extended curriculum script depended on exploiting the distinctive affordance of the dynamic tool to explore how dragging the triangle affected the position of the 'centre'.

This suggests that the teacher's curriculum script was evolving through experience of teaching the lesson with dynamic geometry, incorporating new mathematical knowledge specifically linked to mediation by the software. Indeed, he drew attention to a striking example of this which had arisen from his question to the class about the position of the 'centre' when the triangle was dragged to become right angled:

Teacher: What's happening to the [centre] point as I drag towards 90 degrees? What do you think is going to happen to the point when it's at 90?

Student: The centre's going to be on the same point as the midpoint of the line.

Teacher [with surprise]: Does it always have to be at the midpoint?

[Dragging the figure] Yes, it is! Look at that! It's always going to be on the midpoint of that side.… Brilliant!

Reviewing the lesson, the teacher commented on this episode, linking it to distinctive features of the mediation of the task by the dynamic figure:

I don't know why it hadn't occurred to me, but it wasn't something I'd focused on in terms of the learning idea, but the point would actually be on the mid point.… As soon as I'd said it I thought "Of course!" But you know, in maths there's things that you just don't really notice because you're not focusing on them. And… I was just expecting them to say it was on the line. Because when you've got a compass point, you don't actually see the point, it's just a little hole in the paper… But because the point is actually there and quite clear, a big red blob, then I saw it was exactly on that centre point, and that was good when they came up with that.

In effect, his available curriculum script did not attune the teacher to this property. One can reasonably hazard that this changed as a direct result of this episode.

TIME ECONOMY

Assude (2005) examines how teachers seek to improve the 'rate' at which the physical time available for classroom activity is converted into a didactic time measured in terms of advance of knowledge. The adaptation and coordination of working environment, resource system, activity format and curriculum script are very important in improving this didactic 'return' on time 'investment'.

In respect of this time economy, a basic consideration of physical time for the teacher in this study was the proximity of the new computer suite to his normal classroom:

I'm particularly lucky being next door… If I was upstairs or something like that, it would be much harder; it would take five minutes to move down.

However, a more fundamental feature of this case was the degree to which the teacher measured didactic time in terms of progression towards securing student learning rather than pace in covering a curriculum. At the end of the first session, he linked his management of time to what he considered to be key learning processes:

It's really important that we do have that discussion next lesson. Because they've seen it. Whether they've learned it yet, I don't know… They're probably vaguely aware of different properties and they've explored it, so it now needs to be brought out through a discussion, and then they can go and focus on writing things for themselves. So the process of exploring something, then discussing it in a quite focused way, as a group, and then writing it up… They've got to actually write down what they think they've learned. Because at the moment, I suspect… they've got vague notions of what they've learned but nothing concrete in their heads.

A further crucial consideration within the time economy is instrumental investment. The larger study from which this case has been derived showed that the ways in which teachers incorporated dynamic geometry into classroom activity were influenced by their assessments of costs and benefits. Essentially, teachers were willing to invest time in developing students' instrumental knowledge of dynamic geometry to the extent that they saw this as promoting students' mathematical learning. As already noted, this teacher saw working with the software as engaging students in disciplined interaction with a geometric system. Consequently, he was willing to spend time to make them aware of the construction process underlying the dynamic figures used in lessons:

I very rarely use Geometer's Sketchpad from anything other than a blank page. Even when I'm doing something in demonstration… I always like to start with a blank page and actually put it together in front of the students so they can see where it's coming from.

Equally, this perspective underpinned his willingness to invest time in familiarising students with the software, capitalising on earlier investment in using classical tools:

That getting them used to the program beforehand, giving a lesson where the aim wasn't to do that particular maths, but just for them to get familiar with it… was very helpful. And also they're doing the constructions by hand first, to see, getting all the words, the key words, out of the way.

As this recognition of a productive interaction between learning to use old and new technologies indicates, this teacher also took an integrative perspective on the 'double instrumentation' entailed. Indeed, this was demonstrated earlier in his concern with the complementarity of old and new as components of a coherent resource system.

CONCLUSION

This analysis of a lesson incorporating dynamic geometry illuminates the influence of the key structuring features of working environment, resource system, activity format, curriculum script and time economy on technology use. Although only employing a dataset conveniently available from earlier research, it starts to show the complex character of the professional adaptation on which technology integration into the classroom practice of school mathematics depends. This points to the value of conducting further studies in which data collection (as well as analysis) is guided by the conceptual framework developed in this paper and its predecessor. Such studies might profitably focus not just on the teacher/classroom level, but on the school/departmental level, and the systemic/institutional level.

NOTES

[1] The point at which the perpendicular bisectors of the sides of a triangle meet is the 'circumcentre'. However, in the course of the interview, the teacher referred to this centre as the 'orthocentre'. Note that it is now many years since reference to these (and other) terms – which distinguish the different 'centres' of a triangle – was removed from the school mathematics curriculum in England.

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