# Contents

Introduction

Data-logging and modelling have an established place in the National Curriculum in Science, IT and D&T. New forms of portable technology facilitate capturing data in ordinary classrooms and make it more practicable to work with real data in mathematics, as well.

One of the first design products of the original Microelectronics Education Project (MEP) in the early 1980s was a portable data-logger, called the VELA. This was a large, robust device specifically designed for use in the environment to capture data, such as temperature change, which could subsequently be downloaded to a desktop computer. Since then there have been numbers of educational suppliers and agencies who have developed their own data-capture systems, usually consisting of a control-box, a set of probes, a computer interface and suitable software to download, analyse and display the data. Needless to say the power and price of such systems are quite varied, and there are often compatibility problems in making the data available in a format suitable for use with other software, such as a spreadsheet.

Most of the attention on the use of data-logging in the curriculum has been focused on those subjects which make explicit references to it – i.e. science, ICT and D&T. Within these subjects there are also references to the word 'modelling'. The word usually tends to take one of two meanings! In one case it is used to mean the development of a hypothesis to explain the connection between variables in a system. Thus, for example, an experiment may involve the measurement of the pressure of a particular gas being compressed in a cylinder at various volumes. A phrase such as "*the smaller the volume, the greater the pressure*" expresses one model of the relationship between the volume V and pressure P. Another, such as "*when you multiply the volume V and the pressure P together you get a constant result*", is a stronger statement which, if true, allows you to predict the pressure P, say at any given volume V. In a vaguer sense the word "modelling" is used synonymously with "*What if ...?*", to imply that the data are presented in some numerical form in which the effect of changes can be simulated, or 'modelled' – for example "*do different gases produce the same constant for PV?*". So we have the idea of a 'qualitative model', which is usually a statement in words, but could also be shown in a rough sketch, leading to a 'quantitative model', which is really an algebraic statement, but which could also be represented graphically. In science, once such hypotheses had been tested sufficiently to remove virtually all doubt they used to be referred to as 'laws', as in 'Boyle's law'. Note that such a model, or law, only relates to the chosen variables and does not tell you, for instance, whether the gas is poisonous or not. Such a 'law' may be purely empirical, based just on experimental data, or it may have a theoretical basis in science, such as the Kinetic Theory of gases.
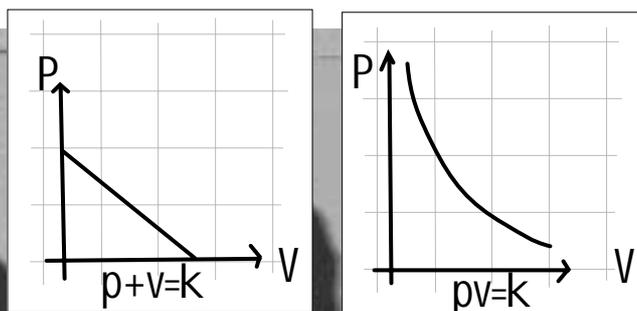
Within National Curriculum mathematics there is considerable scope for the incorporation of work in data-capture and modelling. These particularly relate to ideas of functional relationships, and graphs, in the Algebra strand, and in processing and interpreting data in the Handling Data strand. This book aims to show how teachers of both mathematics and science in a group of schools have been able to work together to help integrate the ideas of data-capture and modelling in planning their lessons. Some of the material was presented at the ASE (Liverpool), MA (Warwick) and ATM (Ripon) annual conferences in 1998.

The project

The Mathematics Curriculum IT Support group was funded by the Department for Education and Employment to help promote the use of Information and Communications Technology in the mathematics curriculum to enhance teaching and learning. As part of its work it supports small-scale, school-based projects which seem to offer interesting models of working which should be relatively easy to introduce into other schools. For 1997 – 98 the project has been based in a group of four schools in central southern England where the mathematics departments have shown interest in developing modelling ideas based on experimental data collected by pupils in science, mathematics and other lessons.

All the schools are LEA 11–16 comprehensives. Two of the schools are inner city, and one of the schools is a girls' school. Two of the schools had already been involved in planning some joint work between mathematics and science departments. One of the schools was already participating in an LEA project on Modelling in Science. All the schools had been involved in developments in IT in mathematics and/or science under the GEST scheme between 1994 and 1996. Each school's mathematics department already had a set of graphing calculators and the project was able to loan each school, where needed, appropriate compatible equipment for data-logging, for whole class display with an OHP and for connection to computers (and hence printers, e-mail and the Internet).

Thus, for simplicity of management, each school had an identical range of equipment at its disposal. However, the approaches used, the activities trialled and the lessons learnt are, to a greater or lesser extent, hardware independent. Thus, for example, where a graphing calculator was used to draw a scattergram, the same data could equally have been imported to graphing software such as *Coypu*, to a statistics package such as *Data Desk*, or to a spreadsheet such as *Excel*.

The data came from experiments performed by the pupils themselves. Some of this they were able to collect using simple measuring devices such as tape measures, thermometers, stop watches and balances. But another objective of the project was to integrate some of the data-logging processes of the science, and other relevant curricula, with the processes of data-handling, transformation, display and analysis associated with mathematical modelling. The actual technology used is described in the next section. Here the aim was to evaluate the potential of relatively low-cost technology to help improve (a) the use of ICT in the mathematics, science and related curricula, (b) the continuity of experience of pupils between the subjects, and (c) the cooperation between the teachers of the subjects.

The form of data-logger which proved simplest to use, the most effective vehicle for achieving the aims, and the most popular with students and teachers was an ultra-sound range-finding device which acted as a 'motion detector' for generating distance-time, and related graphs. At the time of the project these motion detectors were still under development, and were not commercially available – so the range of existing material to support their use was extremely limited. Thus it was most important that the schools should be able (a) to communicate rapidly with each other to share ideas, materials etc. which they generated and (b) to access the most up-to-date information and materials as they became available on the Internet.

Through phase II of its Multimedia Portables for Teachers Initiative, NCET provided teachers from the two Hampshire schools with laptops, printers, modems and Internet access (including e-mail). The Portsmouth schools and the other members of the project team also had access to the Internet and e-mail – so the project team were able to exchange news, experiences, draft materials etc. rapidly, and without the need for the usual face-to-face meetings. As the project developed so it became apparent that there was more scope still for curricular integration involving subjects as diverse as geography, D&T, ICT, PE and PSE. Some of these were explored by staff in the project schools, others remain as fertile ground for others to explore.

As a consequence of the project the Teacher Training Agency (TTA) is supporting a project to develop materials for school-based in-service work to help secondary school mathematics, science and other departments work more closely together using ICT tools in data-capture and modelling.

Since the project finished, NCET has been replaced by Becta. One of Becta's major functions is to oversee the development of the National Grid for Learning (NGfL), and, in particular, the Virtual Teacher Centre (VTC). It is expected that materials such as this book, will be available electronically at Becta's Web site – and that INSET materials, such as those being developed by TTA, will be available through the VTC.

Top: A Y8 science lesson indoors
Bottom left: A Y10 science lesson outdoors
Bottom right: Admiral Lord Nelson School

The equipment

The equipment used in the schools consisted of:

- data-capture devices equipped with a range of sensors

- a special purpose data-capture device known as a "motion detector"

- graphing calculators

- external liquid crystal display panels for projection using OHPs

- cables and software for linking data-loggers and graphing calculators to computers

- computers, modems and printers

- materials and software to support the data-loggers

- connections to the Internet and to e-mail.

Data-loggers

A typical data-capture device, or data-logger, is a microprocessor-controlled object which can store readings from one or more attached 'probes' or 'sensors'. Examples are probes for light intensity using a photo-sensitive resistor, and for temperature using a thermistor. In each case the physical units such as light intensity or temperature are converted into equivalent electrical units such as resistance or voltage. Any such 'analog' quantity is then converted into "digital" data by an 'A-D convertor', which is an integrated circuit (or 'chip') inside the data capture device. Readings taken in this way can be stored in the Random Access Memory (or "RAM") of the device. The device also contains a clock which is used to control the intervals between successive recordings of readings from each probe. As well as accepting inputs from the probes, and storing them in RAM, the data-capture device must be controllable by the user (e.g. to control the time between readings, and the total number of readings to take), and able to output the stored data to another device, e.g. a computer or a printer. The control

might take the form of physical button presses or a set of commands in some control code sent from an external device, such as a computer. The output of the data may be simultaneous with its capture – in which case it is called "real-time data-capture" – or it may be transferred at any time after the readings have been taken.

The version of data-logger used in the project was the Texas Instruments' Calculator Based Laboratory (CBL). This comes complete with sensors for voltage, temperature and light intensity, as well as a cable for connection to calculators. It has input sockets for up to 4 sensors which can collect analog data simultaneously. One of these sockets, called the 'sonic port', can receive data from a range-finding device, such as the CBR described below. It also has ports for 4 digital output and input lines, which have potential for use in control technology applications, such as are met in the D&T and IT curricula. It has an Input/Output (I/O) port for exchanging data with a graphing calculator or computer. Although it is normally driven by 4 AA batteries, there is also a socket for connection to a DC external power source. As well as the three standard probes there is a wide variety of other probes from instrument manufacturers which can be connected. For this project two of the schools also had a variety of probes from the catalogue of the USA Vernier company suitable for experiments in physics, chemistry and biology. One such probe is a simple heart-rate monitor which clips to the ear-lobe, and measures the light intensity – which varies as the pulse pushes blood through a vein.

Motion detectors

These emit an ultra-sound pulse at given intervals, and measure the time taken by each one to be reflected from the nearest object in their 'field of view'. This is the sort of sensor used in the automatic range finder of a camera, and works on the same principle as a bat navigates. They can be connected to a data-logger e.g. through the Sonic input port of the CBL.

The version of the motion detector used by the project schools was one which can be used as a stand-alone special purpose data-logger, called the Texas Instruments' Calculator Based Ranger (CBR), and which can communicate with a graphing calculator directly. This comes with cables for connection

to the CBL, and to a calculator, as well as a clamp for attaching it to objects such as a door frame, or the upright of an OHP.

Graphing calculators

These are really hand-held, battery-powered computers with an integral LCD display. In their ROM they contain a range of mathematical tools for graphing, statistical display and analysis. They contain a programming language and sufficient RAM to store large amounts of data and/or programs. Many models have I/O ports which allow for data interchange between calculators, and also to data-loggers and computers.

The project schools each had a variety of TI-82 and TI-83 graphing calculators, in sufficient quantities to equip a whole class with at least one between each pair of pupils. These models have I/O ports and come with a simple cable for connecting calculators together. Thus data, and programs, can be shared between calculators.

Whole class display

Each school had at least one teacher's version of the graphing calculator which is identical to the student version, except that it has a socket for connection to a large, clear LCD pad, known as a "viewscreen", which sits on top of a conventional overhead projector (OHP) unit, and which projects a copy of the calculator display onto a screen, whiteboard or wall.
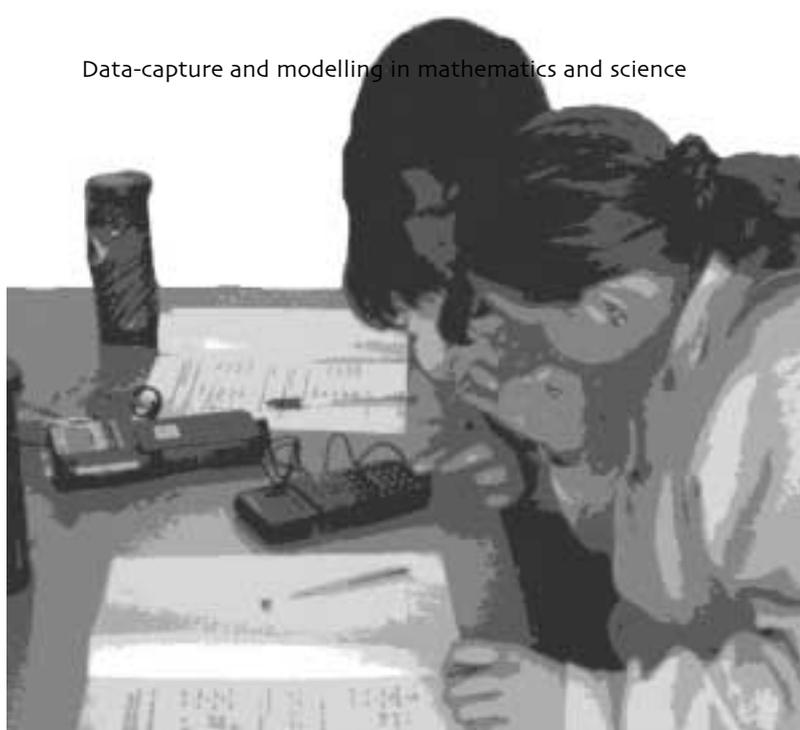
Computer links

Each school had access to *Windows* based PCs. They were given copies of Texas Instruments' GraphLink, which consists of a cable for connecting the serial port of the PC with the I/O port of the graphing calculator, together with the software to facilitate exchange of data, capture of graphic images etc.

Hard copy

With a printer attached to the computer, students can obtain hard-copy of the contents of their graphic calculators e.g. print-outs of data sets, and copies of screen images such as graphs. Using the 'clipboard' these can also be inserted into word-processed reports, worksheets, manuals etc.

Connections to the Internet and to e-mail

All the project team had access to the Internet and e-mail through a modem from a PC (desk-top or lap-top). Using "attached files" with e-mail, the team were able to circulate draft materials in a standard word-processor format (Microsoft *Word for Windows* V.6). They were also able to download software from the Internet to the PC, send it to the graphic calculators and use it to set up the CBL and CBR data-loggers.

(Note: Suppliers' addresses and their current prices are given on page 20.)

In order to facilitate the use of the data-loggers it proved very helpful to have access to pre-written programs for the graphic calculators to set them up for particular experiments. As well as the programs it was helpful, too, to have teachers' notes about the experiments and pupils' materials. One source was published books, which usually contained a disk with the appropriate set-up programs, and sometimes the word-processor files used to make the pupils' materials. Another source was information provided on various Web sites on the Internet.

Above: A laptop, TI-83, CBR and CBL

Distance-time graphs

Each school used the CBR motion-detector, in both science and maths lessons, attached to a teacher's TI-82 or 83 graphing calculator, projecting via the 'Viewscreen' LCD pad on top of a standard OHP. The CBR contains a program in its own memory called 'Ranger' which can be easily downloaded to any compatible graphing calculator. This enables easy control of the CBR and allows a variety of styles of use. Here a mathematics teacher from one of the Portsmouth schools (a girls' school) describes some of the approaches he has used. They illustrate nicely how he managed his lessons around a minimum provision of technology, and also how he built progression into the different activities he planned.

"At my school, distance-time graphs are introduced in Y8/9 depending on the abilities of the groups and so the following work was developed mainly for use at KS3. My main aims for the CBR activities were to try to address some of the issues concerning understanding of graphical interpretation as outlined above using activities that promoted discussion and provided time for reflection. Brian Hudson (Micromath 13(2) 1997) describes this as a 'cycle' of: '...observation, reflection, recording, discussion feedback (test)'. The first set of activities are using the 'Distance Match' option from the 'Applications Menu'. I found that this was suitable for intermediate and higher groups in Y8. With a higher Y9 group the second set of activities are using the distance-time graphs from the plot menu and the third set involved using the functions of the graphical calculator to fit equations of lines to parts of the distance-time graphs.
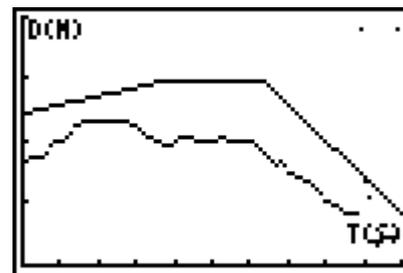
An example of a Distance Match graph: the third screen shows the target graph – with straight lines – and a pupil's attempt to match it

"This activity was started by showing on the OHP one of the graphs from the 'Ranger' program that fitted the constraints of the room. There was then classroom discussion about what the graph could mean and what it represents. We examined the axes and looked to see if we could work out the coordinates of what were considered by the pupils to be the key points of the graph. The class were then split up into groups and given fifteen minutes to try and work out a journey that would correspond to the graph shown. They were each given an A3 piece of squared paper so that they could if they wished sketch the graph. They put a diagram of the journey onto a piece of sugar paper. At the end of the fifteen minutes, one of the groups volunteered to show their conclusions and to map out the journey in the classroom. The CBR was then used so that one of the members of the group could walk through the journey to see how close their match was. There was then a class discussion about how successful the attempt had been and how the journey could be changed in order to more closely match the graph on the OHP. The class were then given another five or so minutes to review their ideas in the light of what they had seen and then another group volunteered. After between 2 and 3 attempts, quite a successful match was made. The pupils were then encouraged to write about their experiences and how their ideas had changed throughout the lesson and how they had changed their initial ideas to get a closer match.

"For the second set of activities the 'Plot' menu was used. Working in groups of 5, the pupils started by devising a journey which they mapped out on the floor of the classroom, taking measurements as necessary. Then, on a piece of A3 graph paper using the same axes as in the 'Plot' menu, they sketched as accurately as they could the graph of what the journey would look like on a distance-time graph. They needed to consider not only the distances involved but also the time needed to travel each part of the graph. When their graphs were ready, after about 30 minutes, the groups took it in turns to explain about their journeys, show their graphs and then, after some rehearsal, to plot their journeys using the CBR. There was then some classroom discussion about how close the OHP graph was to the sketch that the group had produced and the good points of the sketch were picked out by the pupils and ideas were offered about how the sketch could be improved. This activity ran over 3 lessons and during that time the groups were increasingly more able to recreate their journeys on the OHP.

"The third activity again used the 'Dist Match' application. One of the graphs was chosen and shown on the OHP. Each pupil then wrote a report about the graph, describing it as a journey as accurately as they could. Meanwhile I copied the time-distance data stored in the two lists L_1, L_2 from the OHP calculator onto 8 other graphical calculators using the cables and the transfer function. There was then a classroom discussion about the graph on the OHP and the key features were agreed on by the class. They were then split up into groups of 4 so that they could work on a copy of the graph that I had loaded into their calculators. I showed them on the OHP how to do a graph fit using the 'Y=' functions and set the pupils the task of finding the equation for the first part of the graph. They were given about 5 minutes to get as close as they could and then volunteers put their ideas onto the OHP. There was classroom discussion about how close each attempt was to the original graph and a consensus was reached about the 'correct' equation. Similarly the equations of the other parts of the graphs were worked out. During the next lesson, the activity was repeated using a different 'Dist Match' and the equations of the lines were compared. The pupils then set out to find out how they could set up a system so that they could describe lines, i.e. how steep they were etc. Once the pupils had written a journey in the form of a set of equations, they were encouraged to devise a journey which they mapped out in the classroom and to work out the journey in the form of equations only. Each group then set the rest of the class the task of sketching the graphs from the equations that they gave them and then of translating the graphs into a journey. Whilst the rest of the class were engaged upon this task, the group setting the problem were using the 'Plot' menu to produce their own graph using the CBR and then were encouraged to see how closely their equations-based graph matched the one produced by the CBR."

It is important to note here a constraint imposed by the technology. While the axes used in the 'Ranger' program are labelled as T(S) and D(M) for "time in seconds' and 'distance in metres', when an equation is entered on the graphic calculator it must be of the form Y = f(X) e.g. Y = 0.65 + 0.38 X , and so Y here stands for the distance D and X for the time T.

Other teachers preferred to prepare an OHP foil with examples of, say, 5 possible distance-time graphs and to get the students to try to match each one using a CBR and a graphing calculator in groups. Using say 5 CBRs and calculators, it was possible to organise groups of 6 to perform the movements on the playground or in the corridor.

Meanwhile, in a Y11 maths class, the students had also spent a lesson on data-collection and qualitative fitting. This was followed by a lesson on quantitative work in trying to decide suitable linear equations to 'fit' the distance-time graphs. Then they were asked to think of other shapes of mathematical curves they knew, and to try to 'walk' the


(a)　　　　(b)　　　　(c)　　　　(d)　　　　(e)
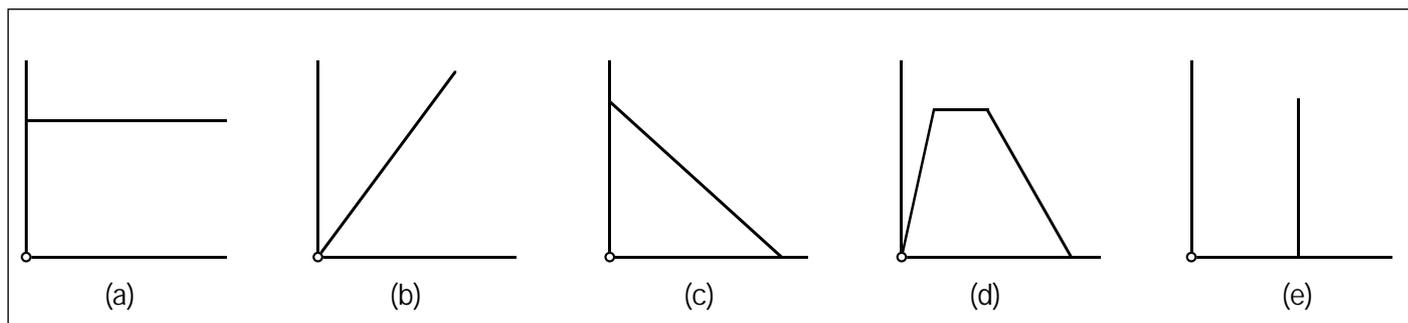
For example, in a Y10 science lesson the pupils were given a variety of different shapes of (piecewise-) linear distance-time graph to try to reproduce: (a) horizontal, (b) diagonal upward, (c) diagonal downward, (d) a mixture of diagonal upward, followed by horizontal, followed by diagonal downward and (e) vertical. Groups of 7 or 8 shared a TI-82 and a CBR, and were let out into the playground to try to 'walk' appropriate journeys, and to keep notes. Subsequently they were asked to write up their observations back in the laboratory, and to describe the velocity at different points on each distance-time graph. The vertical line generated a great deal of discussion, and some very creative thinking!

appropriate trip. Screen 1 below shows one group's attempt to 'walk a parabola'. The noisy data points in the middle are due to the limitations on the CBR's operating range – between a maximum of about 6m and a minimum of about 0.5m. With this data projected, the teacher then got the class to work at finding a quadratic function which gave a good fit to the data points. At first they tried fitting a quadratic in the form $y = ax^2 + bx + c$, but then it was suggested that, since its minimum was around (4,0) a curve of the form $y = k.(x - 4)^2$ might be easier to work with.
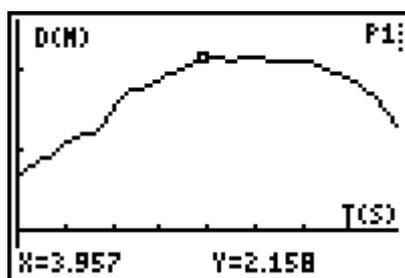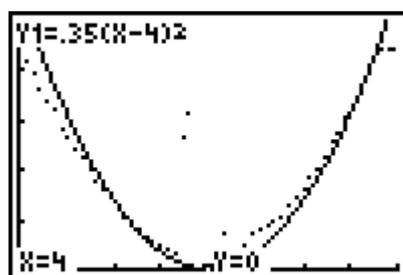

Raw data from a journey
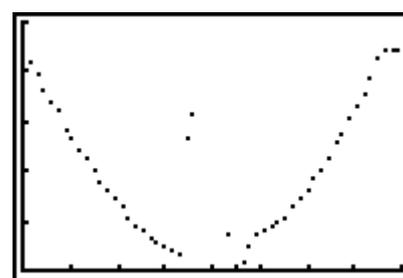

Screen 1
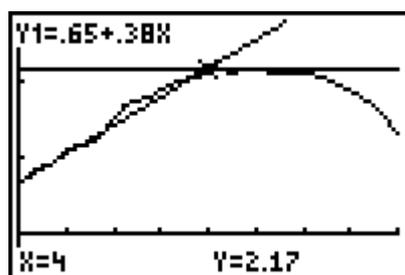
Some attempts to fit functions

Screen 2

Other measurements with the CBR

Obviously both teachers and pupils enjoyed activities using their own movements (*kinaesthesis*) to record distance-time graphs, and to go on to analyse them. In one school data were gathered from pupils running in a PE lesson and the comparison of slopes of the graphs was used to compare the pupils' speeds. But the CBR can also be used easily in other situations which can produce data which are well matched by standard mathematical functions.

For example, the 'Ball Bounce' application in the 'Ranger' program allows you to capture the distance from a bouncing ball to the ground against time. The CBR can be detached from the calculator while performing the experiment. Person *A* stands, perhaps on a chair, holding the CBR pointing towards the ground, while person *B* holds a ball (such as a basket ball) at least 0.5m below the CBR. On the word 'go', *A* presses the CBR's trigger button and B releases the ball. It may take two or three attempts to get the ball bouncing more or less vertically. Then the CBR can be reconnected to the calculator for the captured data to be downloaded and displayed. Obviously it is very important to emphasise (a) that the graph is about motion in one dimension, and is not an *x-y* graph, and (b) that the data have been transformed so that measurements are taken from the lowest point reached (i.e. the floor), rather than the distance from the CBR. The graph is a rich source of modelling opportunities, e.g. describing how the maximum height for successive bounces is decaying.
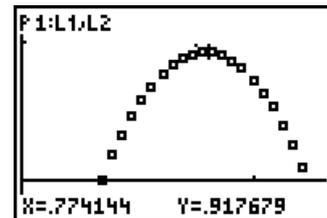
The distance-time graph for each bounce should theoretically be a parabola. The tools available in the 'Ranger' program allow you to extract data for any portion. On exiting from the program the time and distance data are stored, and can be replotted.

Again a model can be found 'by eye'. If the maximum point is at (p, q) then the graph has the form $y = q - k.(x - p)^2$, remembering again that y stands for the height in metres and x for the time in seconds.
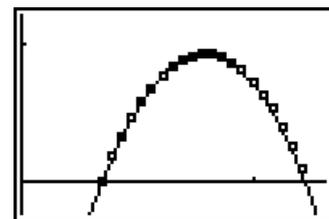
Estimating the velocity should produce a linear function – with a zero corresponding to the maximum height of the bounce. Estimating the acceleration should produce a (negative) constant from which a value of $g = 2k$ can be approximated experimentally. In this example the data are very well fitted by a parabola for which $k = 4.9$, giving the remarkable estimate of g as 9.8 ms^{-2}.

The graphing calculators also contain many powerful statistical functions. With the data stored in lists L_1, L_2 say, a variety of regression models can be fitted automatically. These include linear, quadratic, cubic, quartic, power, exponential, logarithmic, logistic and sinusoidal. Although both the theory, and most of the functions, are outside the National Curriculum, most schools introduced the idea of using linear and quadratic regression as a useful tool to fit functions to number patterns generated in investigations.
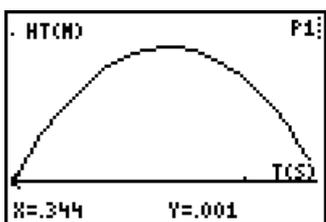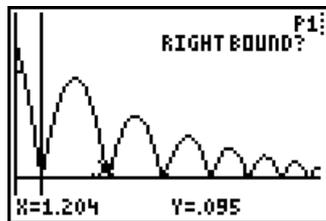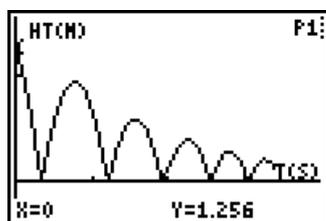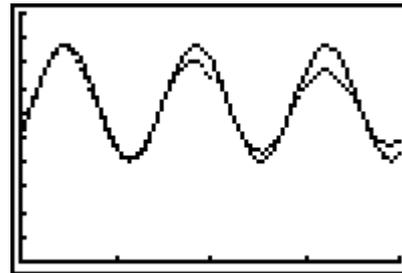
```
QuadReg
 y=ax²+bx+c
 a=-4.703174768
 b=7.388403531
 c=-1.982524085
 R²=.9999875216
```

```
Plot1 Plot2 Plot3
\Y1=0.92-4.9(X-0
.78)²
\Y2=0.92-4.7(X-0
.79)²
\Y3=■
\Y4=
\Y5=
```
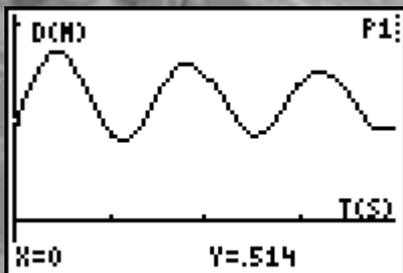
```
Plot1 Plot2 Plot3
\Y1=0.64+.23sin(
4.57X-.58)
\Y2=
\Y3=
\Y4=
\Y5=
\Y6=
```


As an introduction to trigonometric functions the CBR can be set up under a spring-mass system, or in the extended path of a simple pendulum.

```
MAIN MENU     ▶START NOW

REALTIME:  NO
  TIME(S):  4
 DISPLAY:  DIST
 BEGIN ON:  [ENTER]
SMOOTHING:  LIGHT
   UNITS:  METERS
```

```
 D(M)              P1
 ___
/   \    ___    ___
     \__/   \__/   \
                    T(S)
X=0           Y=.514
```

Once again, a 'by eye' fit can be made by transforming the graph Y=sin(X) to the form Y = A + Bsin(CX + D).

Of course, we have only fitted a model to just a portion of the first wave. Clearly it looks as if the amplitude is being damped – and so the amplitude a should itself be a function of t (and we would expect it to be negative exponential) – but that's A-level! The nice thing is that the same ICT technology and skills can be used in a variety of levels of activity which demonstrate progression. In the words of Jim Kaput, the use of this kind of hand-held technology is "democratizing access to the mathematics of change".
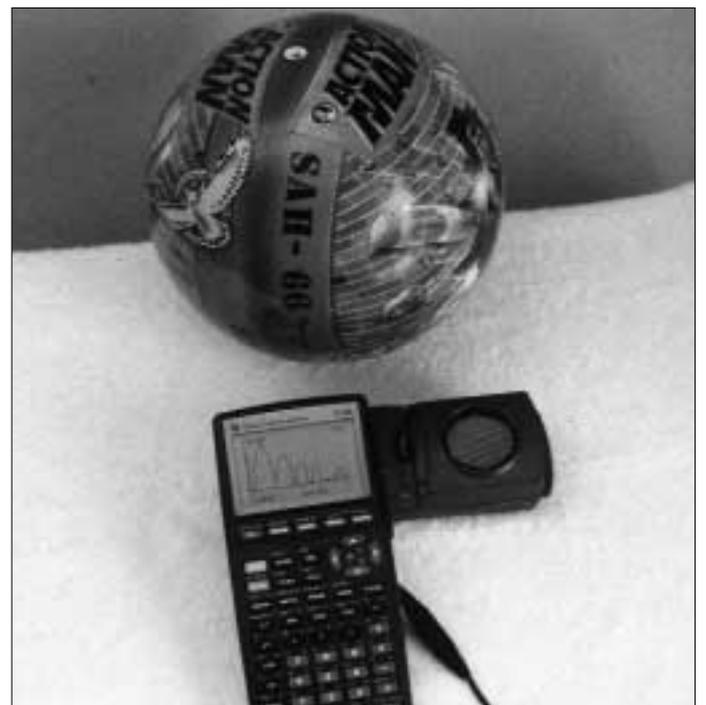
Above: All the equipment needed for 'Ball bounce'

Pupils' and teachers' reactions

Here are some pupils' comments from one school on how they found using the CBR for practical work in the classroom had helped their learning.

Sebastian Day   Maths Lesson   2.10.97

The main thing about the lesson wasn't what I learnt but what I will remember. Because of the lesson I doubt that I will ever forget the equation for a line.

$$y = m x + c$$

Nor will I forget what each of the letters stand for.

Thinking about the lesson, having some one walk up and down the classroom made it easier to understand the graph and de-code the information that was given to you.

But now because of this, when ever I see a complicated graph All I have to do is imagine myself walking the same as the graphs lines and then I can understand exactly whats happening.

Another thing that I learnt that I'm sure I'll find useful is that the gradient for a line on a distance time graph is the same as the speed.


Russell Labibi   Maths Homework
3.10.97

The lesson worked well and it was a good way of teaching something which otherwise could have been boring. I learnt a lot about distance-time graphs. I learnt about accelaration and velocity. The equation y = m x + c is used to draw lines on graphs. y stands for the y axis, m for the gradient, and c for the intercept.

The intercept can easily be read off of the graph. To work out the gradient you do the distance travelled divided by the time travelled. The graphic calculators were also good, keeping the class interested. It was also a good way of teaching us how to find gradients and intercepts.

Terry Ward 1O HW

In maths between 29/9/97 and 3O/9/97 we were doing work with a graphic calculator. In the first lesson we were using sensors to learn how a graphical calculator plots a distance-time graph.

We had to move forward or backwards to try and match a graph on a wall. The sensors detected how far backwards or forwards we travelled and plotted it against the graph displayed on the white board e.g.
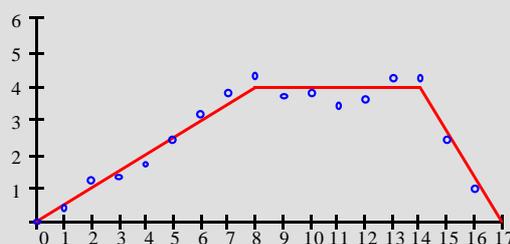
On the second lesson sir displayed a graph on the white board and then handed out some graphical calculators.

The graph was displayed on each calculator and our job was to enter the coordinates to match the graph. After a while we found out the equation by trial and improvement. Sir told us how to find the intercept without an intercept point and we also re-learnt the equation for a straight line.

All the teachers in the project found the equipment extremely reliable and easy to use. Obviously the sensitivity of the CBR, and its range of 0.5m – 6m, restricted some of the ideas that they discussed for practical work – for example it wasn't really possible to measure speeds of cars on a road, or easy to pick up a rolling object on the ground. But clearly a good range of practical activities were possible, which previously would have been impracticable to carry out – either in science or mathematics. The teachers were clearly encouraged by the way that the pupils responded to this kind of work.

The teacher from the Portsmouth girls' school summed up his reactions by:

> *"The use of the CBR and the graphical calculators was very popular with the girls. By monitoring the discussions and the interactions*

during the activities I observed many cycles of 'observation, reflection, recording, discussion feedback (test)'. The technology used in this way, supporting collaborative learning, empowered the girls to develop their levels of understanding as they tried to fulfil the aims of the activities. I believe that by developing more activities within the framework that I have set out above, using the CBR actively encourages pupils to make the very important links between graphs and algebraic relationships."

He went on to quote from Paul Ernest (1996):

"First of all there is the active construction of knowledge, typically concepts and hypotheses, on the basis of experience and previous knowledge. These provide a basis for understanding and serve the purpose of guiding future actions. Secondly there is the essential role played by experience and interaction with physical and social worlds, in both the physical action and speech modes. This experience constitutes the intended use of the knowledge, but it provides the conflicts between intended and perceived outcomes which lead to the restructuring of knowledge, to improve its fit with experience."

It is interesting to contrast the pupils' responses with those observed by Kath Hart:

"...several pupils found difficulty with travel graphs, even though they appeared to be able to give the correct answer. It was clear that several of those interviewed had incorrect perceptual interpretations of the graph. Some thought of the graph as a journey that was up and down hill, or as directional on the ground, and found it difficult to deal with the abstract notion of distance from the origin. Many had no idea what was intended, while others had very vague ideas about graphs. ...While many children will be able to read information from a graph or to plot given data, it seems only a few will be able to understand the connection between an equation and a graph."

Other teachers reported on the remarkable progress of less able pupils, such as a bottom set Y9, in their

understanding and interpretation of such graphs, even with access to only a single CBR.

This project did not provide the opportunity to work with any A-level classes, but it became obvious that the equipment would permit experiments which would be invaluable in introducing differential calculus through velocity as the instantaneous rate of change on a distance-time graph, and of integration through distance travelled as the summing of areas under a velocity-time graph. These ideas are not new, but the use of this technology makes it much more accessible. A major programme, called SimCalc, is being directed by Jim Kaput of the University of Massachusetts-Dartmouth for the USA National Science Foundation. Its Web site address is given in Appendix 2.

Below: Capturing heart-rate data

Sensing other data – the CBL

The Calculator Based Laboratory (CBL) is much more versatile than the CBR range-finder, but the teachers found that they needed access to materials and software to make it accessible. A list of these is included in the Appendix.

The three standard probes which come with the CBL – for light intensity, temperature and voltage – can be used to capture data from simple experiments which again should give good opportunities for proposing qualitative models, for discussion about rates of change and, perhaps, for proposing quantitative models by fitting standard functions. For example, a cooling curve can be used to compare the behaviour of excess temperature with time, perhaps leading to an exponential fit, as can the discharge of a capacitor across a resistor. The light sensor used against a bulb or neon tube can detect the cyclic behaviour of the AC circuit. A microphone provides another very good source of data e.g. for simple and complex sound waves. The CBL can also collect data fast enough to be able to estimate the speed of sound!

The first simple experiment uses a plastic veterinary syringe filled with air. It is attached to a pressure sensor connected to the CBL. Readings of the pressure are recorded at different positions of the plunger, corresponding to values of the volume at 20, 18, 16, ... cm^3. The data are then entered manually into the statistics lists of the graphing calculator. The list L_1 holds the volume in cm^3 and L_2 holds the pressure in atmospheres. The scatter-plot of L_1 against L_2 clearly shows a qualitative 'the smaller the volume, the greater the pressure' relationship – but not a linear one! If an inverse relation is suspected then a new list L_3 can easily be made by multiplying L_1 by L_2 term-by-term. The values in L_3 seem more or less constant, with an average value of about 19. In fact the calculator has a function which will calculate the mean of any list. Clearly there is scope for experimental error in making the pressure recordings exactly at the given volumes, and the power regression reveals a model of the form $y = k.x^{-0.91}$ rather than $y = k.x^{-1}$.
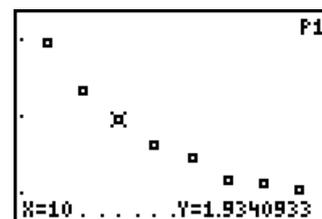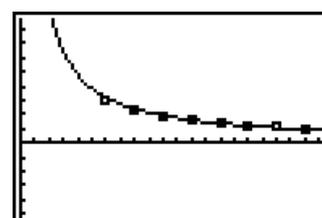
The next example uses the light probe of the CBL to sense the flicker-rate of the fluorescent tube in a photographic light-box. First it is useful to have some pre-written programs to help. One source is the set of CBL programs distributed with the TI-Graph Link software, for which there are also accompanying work-books available, such as TI's 'Physics and Maths with the CBL'. Another source is through the Internet. Texas Instruments' calculator Web site is at www.ti.com/calc, and another useful source is from Vernier Software at www.vernier.com. For this experiment the teacher used the Vernier Software's Data Collection Programs, which can be freely downloaded from their Web site, or obtained on disk (e.g. from Oxford Educational Supplies), or bought together with an accompanying project book, such as their 'Physical Science with CBL' by Donald L. Volz and Sandy Sapatka.

Once the TI-83, CBL and light-probe have been set up, the PHYSICS program does the rest!

```
EXEC EDIT NEW
1:PHYSICS
2:PHZCALIB
3:PHZCALS
4:PHZGRAPH
5:PHZMICRO
6:PHZMONIT
7↓PHZOPTIO
```
1

```
VERNIER SOFTWARE

    PHYSICS
  WITH THE CBL
 FOR THE TI-83

4/14/97   [ENTER]
```
2

```
***MAIN MENU***
1:SET UP PROBES
2:COLLECT DATA
3:VIEW GRAPH
4:RETRIEVE DATA
5:OPTIONS
6:QUIT
```
3

```
ENTER NUMBER OF
PROBES:1
```
4

```
SELECT PROBE
1:MOTION
2:FORCE
3:ACCELEROMETER
4:MICROPHONE
5:PRESSURE
6:TEMPERATURE
7:MORE PROBES
```
5

```
SELECT PROBE
1:LIGHT
2:MAGNETIC FIELD
3:VOLTAGE
4:C-V CURRENT
5:C-V VOLTAGE
6:THERMOCOUPLE
7:MORE
```
6

```
USE LOWEST
AVAILABLE
CHANNELS.

ENTER CHANNEL
NUMBER:1█
```
7

```
DATA COLLECTION
1:MONITOR INPUT
2:TIME GRAPH
3:TRIGGER/PROMPT
4:TRIGGER
5:RETURN
```
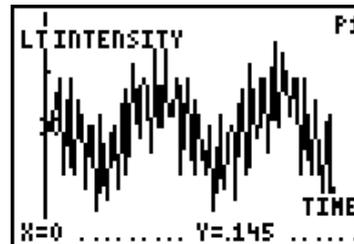8

```
ENTER TIME
BETWEEN SAMPLES
IN SECONDS:.0002

ENTER NUMBER
OF SAMPLES:100
```
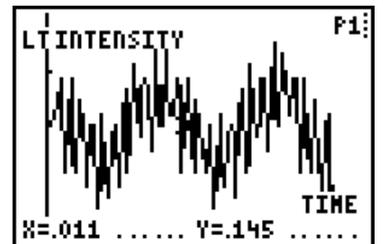9

```
READY EQUIPMENT.

PRESS [ENTER] TO
BEGIN COLLECTING
DATA.
```
10

```
LT INTENSITY               P1
                          TIME
X=0 ........ Y=.145 ......
```
11

```
LT INTENSITY               P1
                          TIME
X=.011 ...... Y=.145 ......
```
12

With the graph on the display it is possible to move the cursor from side to side to trace along it, and to read off coordinates. This reveals the period of the flashing to be about 0.01 sec, equivalent to a flicker rate of about 100 cycles/sec (Hz).

Actually the core of such programs to set up the CBL and to recall data is really quite simple, and this experiment could equally as well have been done with a program keyed directly into the graphing calculator. The CBL actually has a number of built-in data smoothing algorithms which are useful to 'tidy up' the data. Data is exchanged with the CBL using lists.

```
Send({1,0}):Send({1,1,1})
Send({3,.0002,100,0,0,0,0,0,1,5})
Get(L₂):Get(L₁)
PlotsOff :FnOff
Plot1(xyLine,L₁,L₂,·):ZoomStat:Trace
```

'Send' is the command to transmit to the CBL. {1,0} is the command to clear the CBL, {1,1,1} is the command to use the probe in Channel 1, and for the CBL to auto-identify its type. {3,.0002,100,0,0,0,0,0,1,5} is the command to set the sample rate at 0.0002 secs and to collect 100 samples. The string of zeros are parameters which

could be changed to set up how the sampling is triggered. The last two parameters determine if, and how, time is stored in the CBL, and which, if any, of 6 noise filtering algorithms to apply. Here the CBL is set to record the absolute time in seconds, and to apply a '3-point median pruning filter'. 'Get' is the command to receive from the CBL. When the data have been collected by the CBL it will return the stored data from Channel 1 into list L_2, and the recorded time into list L_1. The program could just stop there, and any further analysis and display be undertaken directly on the calculator. The last lines are there just to show how the graphical display can also be controlled by the program. Once the program has finished the lists can be accessed with the STAT Editor.


The light experiment shows how the CBL can record data at very small intervals of time to reveal behaviour which cannot be sensed directly by a human observer. A more extreme example is given by a simple experiment to estimate the speed of sound.

Again this uses Vernier's Physics CBL software. The apparatus used is a 2m long piece of round plastic drain pipe, blocked at one end. The CBL is attached to the I/O port of a graphic calculator (TI-83), and to a microphone in channel 1, using a DIN connector cable. The microphone is placed close to the open end of the pipe, and a sharp sound is made, such as by knocking two screwdrivers together (or a hand-clap will do). The CBL needs to be set up to record sound sufficiently frequently to 'hear' the original 'bang', and then its echo – so this requires some estimation. One of the many clever features of the CBL is the ability to define a 'trigger' value for one of the sensors upon which data recording will start. Another is its ability to 'prestore' a number of readings taken just before the trigger is activated. Again the program makes it very easy to set up. Here the microphone is used to record voltages, and the "ground" value is found to be about 2.5 V.
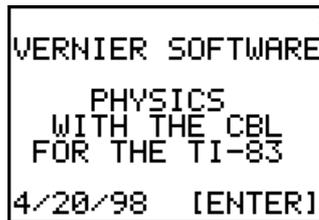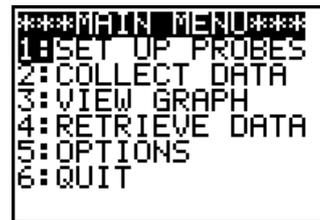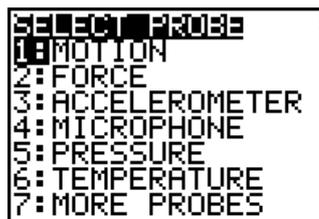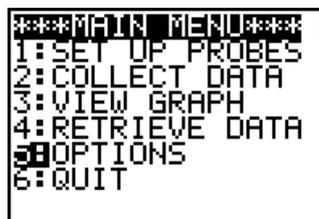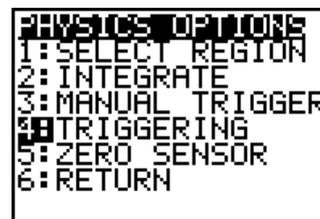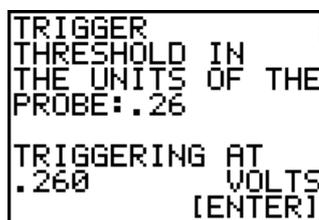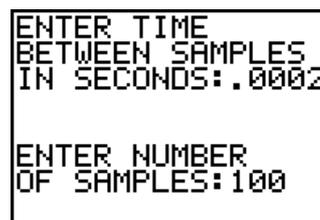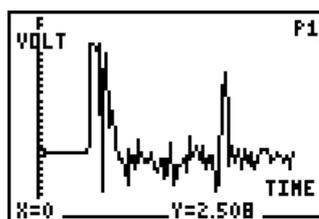
Clearly there are two peaks, the first corresponding to the first sound heard, and the second to its echo. Leaving the program we can access the recorded

data which is stored as times in list L_1 and voltages in list L_2.
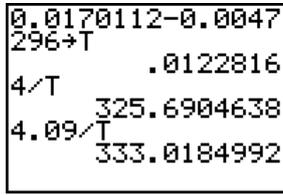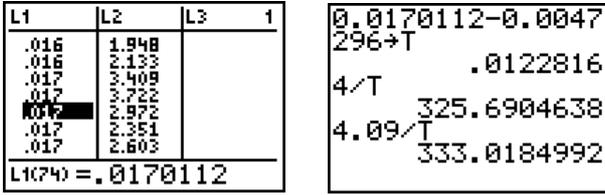
Thus we can estimate that the first 'bang' occurred at time 0.0047296 s., and the echo was received at time 0.0170112 s. – giving an elapsed time T = 0.0122816 s. between events. Using 2 metres as length of tube gives D = 4 m, and D/T = 326 ms⁻¹ as an estimate for S. Most reference books give the speed of sound in air at 0°C as S = 332 ms⁻¹.

But in fact the microphone was placed a little way away from the open end of the tube. Further measurements show it to be 4.5 cm from the end of the tube – so a further 9 cm need to be added to D, which then gives 333 ms⁻¹ as the revised estimate for S!

Why then did a change as small as 0.09 m in D produce a change as large as 7 ms⁻¹ in S? Well, the problem here is the division by T, which is a small number – and has the same effect as multiplying by 1/T, which is about 80 – so that a change in distance of about 0.09 m produces a change in speed of about 7.2 ms⁻¹.

But looking at percentage changes shows that a change in D of about 0.09/4 = 2.25% produces a change in S of about 7.3/325.7 = 2.24%.

Using 3 significant figures for the elapsed time T shows that taking T = 0.0122 secs gives S = 335 ms⁻¹, and taking T = 0.0123 secs gives S = 332 ms⁻¹.

So this gives (a) a practical example for thinking about accuracy of computations and measurements, and (b) a remarkably simple way of making a very good estimate of the speed of sound.

The next two examples are of particular importance in paving the way for the introduction of the exponential function in mathematics. The first uses the CBL's voltage probe to plot the discharge of a 220 μF capacitor wired in parallel with a 100 kΩ resistor across a 9V cell. This uses the CHARGE program for

Activity 12 in TI's 'Real-World Math with the CBL system' by Chris Breuningen *et al*. As the connection with the cell is broken, so the CBL starts recording the voltages across the capacitor at intervals of 0.5 sec for 50 sec.
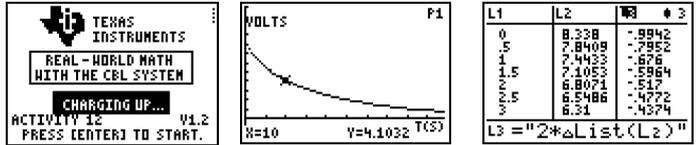
The time in seconds is stored in list L_1, and the voltage in list L_2. Approximations to the discharge rate in volts/sec can be found by dividing the differences between successive voltage samples by the corresponding difference in times (0.5 sec) at which they were logged. This shows the power and flexibility of the list functions of the graphing calculator. By tracing the graph, or moving the cursor along the list L_2 it is easy to estimate the 'half-life' of the voltage decay, and to check whether the time taken to go from 8.34 volts to 4.17 volts is the same as to go from 4.17 volts to 2.08 volts etc.

The example below is a cooling curve obtained with the CBL's temperature probe and the CHILL program from Activity 10 of the same source. The temperature probe is immersed in a hot liquid for a couple of minutes. When the probe is removed from the liquid, the CBL starts recording the temperature at 1 sec intervals for 100 seconds as the tip of the probe cools in air, so the experiment doesn't take very long to complete! Here the program's use of °F reveals its USA roots (at the moment nearly all the curriculum materials are from the USA). Actually it is quite easy to edit the program to gather and plot data in °C, but this serves as an example of the power of the graphing calculator to calculate with lists.
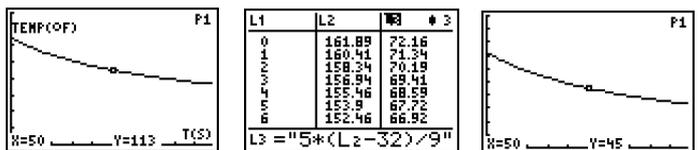
This program has also been adapted for the CBL to work simultaneously with two temperature probes to compare rates of cooling. This was used with Y8 classes to model the situation of 'why penguins huddle together to keep warm', using test-tubes of hot water in different configurations.

Linking with other subjects

Although the primary focus of the project was on mathematics and science, the ideas soon impinged on other areas of the curriculum.

In some schools these were on environmental issues which involved science, geography, PSE, ICT and mathematics. In most of the examples considered so far the sample rates of the data-capture devices (CBR and CBL) were set to be quite short. However, for studies of light and temperature in the environment the CBL was set to record values at hourly intervals over 5 successive days, and the data were then transferred from the graphic calculators to spreadsheets on the school network. In another experiment the light and temperature were recorded throughout the day in a variety of locations around the school. As well as analysing the statistical data in mathematics, the students undertook a project (with a word-processed report) to suggest a range of plants suitable for growing in the various locations. (Incidentally, a high school in the USA had a CBL-based experiment on the effect of weightlessness on the growth of bean sprouts go up in one of the latest American space probes.)

One school based a programme of experimental work on the GCSE 'Health and Fitness' module in PE. The pupils used simple probes to measure light changes in the blood vessels in their ear-lobes to monitor their heart rates before and after exercise.

An apparent curiosity with the CBL is that it has sockets marked 'Dig In' and 'Dig Out', but that there are no materials to suggest how to use them. Texas Instruments in Dallas USA put the project in contact with a physics teacher, Dr Fred Thomas of Sinclair Community College, Dayton, Ohio, who had been working on a small robot, like a floor turtle. This has a pair of independent motors controlled by a small board connected to the 4-line Digital Output port of the CBL. The CBL is also connected to sensors e.g. for sound and/or light. A graphing calculator is attached with Velcro to the top of the CBL – so the whole device is a completely self-contained robot, and runs under the control of a program entered into the calculator.

This idea was the catalyst for getting the interest of colleagues in D&T departments, and ICT coordinators, in the schools. The first step was to adapt a standard educational digital control adaptor (the 'Blake box') for use with the CBL and graphing calculator instead of a computer. This means that, in addition to the data-logging and modelling work, a large amount of feedback and control activities in D&T and ICT can now be done equally on graphing calculators as on computers. However, the real interest now lies in looking at activities which become possible through the free-standing, compact nature of the graphing calculator which could not be undertaken with conventional ICT. An example is the prototype CBME (Calculator Based Mars Explorer – or 'buggy') which the project team built. This has a Meccano chassis, powered by a 6V motor through a gear train. The CBL is connected to a TI-83 graphing calculator, a CBR motion detector, an analogue light-sensor and the 'Blake box' digital interface. The 'Blake box' is powered by the same 6V source as the Meccano motor, which it can switch on, off or reverse with a relay attached to two of its digital output lines. There are also a buzzer and a light bulb attached to another two output lines.

The graphing calculator contains a simple program which carries out this sequence of operations:

● Monitor the light-sensor until a certain threshold is reached (day break on Mars!)
● Then drive the motor forward while monitoring distances recorded by the CBR until an object is located within 1 metre of the buggy (a Martian, or a Mars rock?)
● Then stop and flash the bulb for 5 seconds
● Then reverse the motor for 5 seconds while sounding the buzzer
● And then stop.

Although the technological focus of the work was on data-logging devices, some schools also found other ICT-based activities which helped in linking mathematics, D&T and ICT. Some of the schools already use dynamic geometry software such as *Cabri Géomètre* and the *Geometer's Sketchpad*, both for graphic design in ICT and for work in mathematics, such as on 'Shape, Space and Measures'. They were able to use these to model simple mechanisms, such as slider-crank and 4-bar linkages, in D&T. Here the animation facility is a particularly attractive feature.

Future developments

It became clear in looking for suitable science experiments for the CBL that access to supporting materials was a key point. There are quite a variety of US materials, but not yet any UK ones. A particularly helpful form of materials were a series from Vernier with separate (substantial) books for physics, chemistry and biology – each with disks containing both the graphic calculator programs to set up the CBL, and the Microsoft *Word* files used to create the student work- and lab-sheets. This meant that both the programs, where necessary, and the materials could readily be adapted to the UK context. There is also a wealth of similar electronic supporting materials available for free down-load from Web sites such as TI and Vernier in the USA. This suggests that the take-up of such data-logging work would be considerably facilitated by having materials relevant to the National Curriculum on a UK Web site, such as the Virtual Teacher Centre (VTC) on the National Grid for Learning (NGfL) currently under development by Becta. Ideally, professional associations, such as ATM, MA, ASE etc. could be supported in bringing this about.

In that respect it also became clear how important it was for the project schools to have Internet access and e-mail. Teachers from two of the project schools were given multimedia laptop systems as part of the NCET project and by the end, all those involved in the project were able to send ideas, comments, draft materials etc. to each other by e-mail.

One important aspect of ICT and education nowadays is pupil access outside the school. By chance one of the teachers brought to a meeting some software she had bought for her own children. Called 'Mathsoft StudyWorks!' it is a CD-ROM and small manual which can be bought from stores like W.H.Smith and PC World for around £25. Two items first caught the imagination. On the back of the box

it advertises: 'Compatible with Graphics Calculators!' and 'Connect to the world of maths and science'. The former means that you can easily import data collected from a CBL/CBR and stored on a graphing calculator for display, analysis and/or incorporation in a printed report. You can also drop in bitmap images e.g. from screenshots taken from a calculator. The second means that you can access the Web, and send e-mails, from within the software. But closer analysis reveals far more. The software is a virtually full implementation of the powerful *MathCad* software, including aspects of the computer algebra system (CAS) *Maple* for symbolic manipulation! It is certainly very good value for money – and appears to be an ideal tool for use by teachers and students alike, whether at home or on a PC (or network) in school. So the whole aspect of communication using a range of ICT devices is one which needs more study.

The schools which took part in the project only had a very small amount of external support, but they made extensive use of their own schools' INSET days – often planing collaborative work between subjects. The TTA is supporting the development of materials to support just such inter-subject, school-based activities using hand-held technology. It is to be hoped that the major initiative, funded by the National Lottery, to train all teachers to use ICT to teach their subject(s) will provide an opportunity both for cooperation between subjects, such as science, D&T and mathematics, and for the use of a variety of ICT, including hand-held devices such as graphing calculators and data-loggers.

Appendix 1: **References**

Books

Biology with CBL, Holman, S. & Masterman, D., Vernier 1997, £34.50**

Chemistry with CBL, Holmquist, D., Randall, J, & Volz, D., Vernier 1997, £34.50**

Datalogging and Control, Roger Frost, ASE, £16.50

Exploring experimental data with IT, (4 pp), NCET, 1996, free

Exploring Physics and Math with the CBL System, Brueningsen, C. & Krawiec, W., Texas Instruments, 1994, £19.95*

Math and Science in Motion: CBR Activities for Middle School, Brueningsen, C., Brueningsen, E. & Bower, B, Texas Instruments Explorations, 1997

Mathematics meets Technology, Bolt, B., Cambridge 1991

Modelling Motion: High School Math Activities with the CBR, Antinone, L., Gough, S. & Gough, J., Texas Instruments Explorations, 1997

Modelling with computers in science, (8 pp) NCET 1996, free

Physical Science with CBL, Volz, D. & Sapatka, S., Vernier 1997, £34.50**

Real-World Math with the CBL System, Brueningsen, C., Bower, B., Antinone, E. & Brueningsen, E., Texas Instruments, 1994, £19.95*

Secondary Mathematics with a Graphic Calculator, Oldknow, A., Pope, S., Waddingham, J. & Winbourne, P., in the *IT Math Pack*, ATM/MA, 1994 £15

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Oldknow, A. (1998), 'The calculating kind', *TES* Friday, May 22 pp 20-31

Taylor, R. (1995), 'Using the Graphing Calculator - in Physics Labs', *The Physics Teacher*, 33 (May)

Wake, G. (1996), 'Modelling Motion with Dynamic Geometry Software', *Micromath*, 12(3)

Wills, N. (1997), 'Motion Graphs with the TI-82', *Micromath* 13(3)

* includes disk(s) with CBL programs for TI-82/3
** also includes disk(s) with word-processor files for worksheets etc.
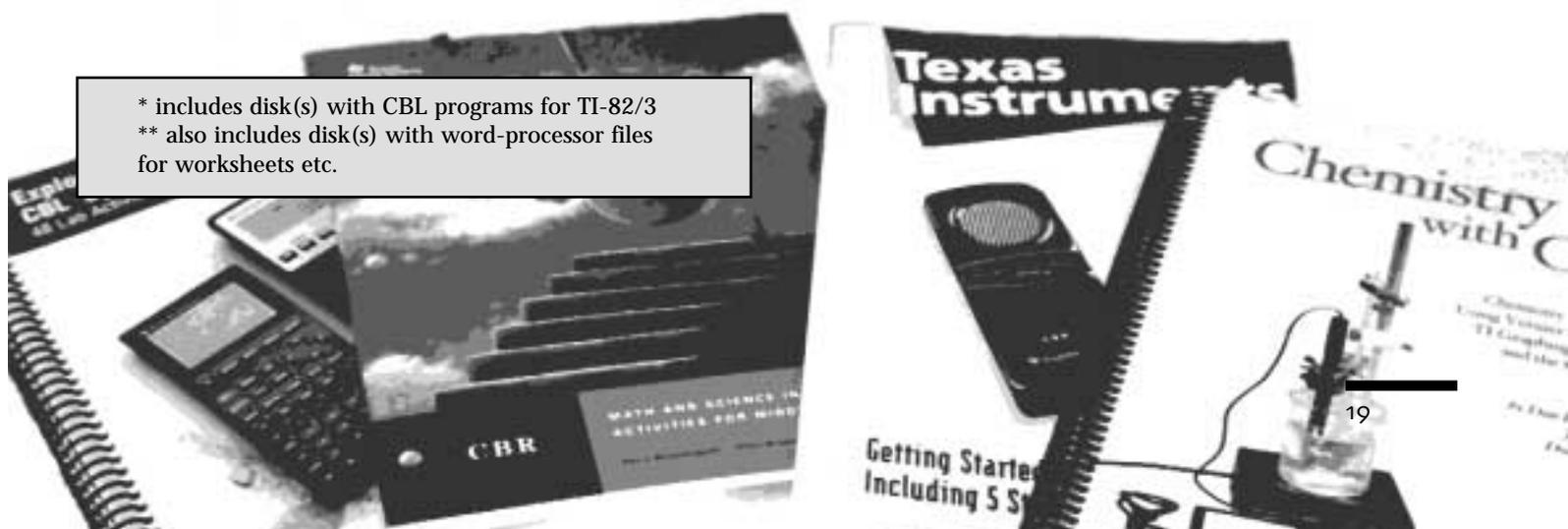
Appendix 2: **Useful addresses**

Materials

Association for Science Education,
College Lane, Hatfield AL10 9AA
(tel 01707 267411, fax 01707 266532)

Association of Teachers of Mathematics,
7 Shaftesbury Street, Derby DE3 8YB
(tel 01332 346599, fax 01332 204357)

British Educational and Communications
Technology Agency,
Milburn Hill Road, Science Park, Coventry CV4 7JJ
(tel 01203 416994, fax 01203 411418)

Hampshire Microtechnology Centre,
The Parkway, 94-96 Wickham Road, Fareham,
Hants PO16 7JL
(tel 01329 519119 fax 01329 316222)

The Mathematical Association,
259 London Road, Leicester LE2 3BE
(tel 01162 703877, fax 01662 448508)

Equipment

The approved supplier (other suppliers keep stocks) for Texas Instruments' products is:

Oxford Educational Supplies Limited, Weston Business Park, Weston on the Green, Oxford, OX6 8SY (tel: 01869 343369, fax: 01869 343654)

Educational (ex-VAT) prices for the equipment used in the project are as follows (correct at the time of going to press):

TI-82 graphing calculator, student version	£ 49.95
TI-83 graphing calculator, student version	£ 59.49
TI-82 teacher display pack: calculator, viewscreen and case	£174.95
TI-83 teacher display pack: calculator, viewscreen and case	£184.95
TI-CBL complete with case, 3 sensors and guide book	£149.00
TI-CBR complete with cables, clamp and guide book	£ 59.95
TI-Graph link with cables and software for 82/83	£ 34.95

Additional probes and sensors from Vernier used:

Microphone	£ 24.50
Heart rate monitor	£ 38.95
Pressure sensor	£ 49.95
Ph system	£ 59.95
CBL-Din adaptor	£ 4.95

Software

Oxford Educational Supplies also stock Vernier's *Graphical Analysis* software (£39.95) for importing data to PC or Mac for analysis and plotting. A demo version is available free from Vernier's Web site, as is a set of programs for use with collecting CBL data for physics, chemistry and biology (also obtainable on disk from Oxford Educational Supplies at £9.95).

Many high street stores, such as PC World, stock a CD-ROM called '*Studyskills Mathworks'* at about £25 – £30. This is a very full implementation of Mathsoft's *Mathcad* software, including *Maple* symbolic manipulation, which will also import data files from graphing calculators.

Books

Oxford Educational Supplies stock the Vernier books on CBL use in physics, chemistry and biology (at £34.50) and the Texas Instruments books for the CBL (at £19.95).

Web sites

Roger Frost:
http://ourworld.compuserve.com/
homepages/Roger Frost/
Micromath:
http://mcs.open.ac.uk/cme/micromath/
Schools OnLine Science:
http://www.shu.ac.uk/schools/sci/sol/contents.htm
SimCalc project:
http://www.simcalc.umassd.edu/
Texas Instruments: http://www.ti.com/calc
Vernier Software: http://www.vernier.com
Virtual Teacher Centre:
http://vtc.ngfl.gov.uk/resource/cits/maths

Mathematics CITS group materials

Dynamic Geometry

Mathematics and IT –
a pupil's entitlement

Mathematics and IT at Work

Primary Mathematics with IT

Review of IT in Mathematics –
a departmental self-audit

All the above available from Becta.

IT Maths Pack
(from ATM and MA)

Maths CITS web site on the Virtual Teacher Centre
of the National Grid for Learning at
http://vtc.ngfl.gov.uk/