DYNAMIC GEOMETRY SOFTWARE: THE TEACHER'S ROLE IN FACILITATING INSTRUMENTAL GENESIS

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In the UK, use of dynamic geometry software (DGS) in classrooms has remained limited. Whilst the importance of the teacher's role is often stated in dynamic geometry research, it has been seldom elaborated. This study aims to address the apparent deficiency in research. By analysing teacher/pupil interactions in a DGS context, the intention is to identify situations and dialogue that are helpful in promoting mathematical thinking. The analysis draws on an instrumental approach to describe such interactions. Elements of instrumental genesis are distinguished in pupils' dialogue and written work which suggest techniques that teachers can employ to facilitate this process.

Keywords*: teacher's role, dynamic geometry, instrumental genesis*

INTRODUCTION

This study aims to elicit teaching techniques that teachers might employ in their classrooms to help pupils engage constructively with dynamic geometry software. Currently DGS has made little impact on classrooms in the UK. Research has tended to focus on elaborating situations of innovative use and student/machine interaction. This study hopes to re-focus on "the teacher dimension" (Lagrange et al, 2003). The author carried out this study in the role of a practitioner-researcher with a high ability year 8 class. Whilst the class cannot be deemed to be representative, nevertheless it is an 'ordinary classroom' and therefore this study can claim to respond to the need for research into how dynamic geometry software is integrated into 'the *regular* classroom' (Gawlick, 2002).

DGS – A CLASSROOM FAILURE?

Dynamic geometry software (DGS) appears to be following the cycle of oversell and high expectations, ending in limited classroom use identified by Cuban (2001) as a general pattern for technological innovation in education. Research in mathematics education generally presents DGS as a potentially important and effective tool in the teaching and learning of geometry (see for example Holzl, 1996; Marrades and Gutierrez, 2000; Mariotti, 2000). In their survey of geometry curricula, Hoyles et al (2001) state that although most countries seek to integrate ICT into teaching geometry, there is little explicit influence of ICT in classrooms. In the UK, despite recommendations in the Key Stage 3 Mathematics Framework (DfEE, 2001) for using DGS to develop geometrical reasoning, classroom use has remained limited (Ofsted, 2004). Syntheses of research findings generally conclude by favouring the strong potential of ICT but give few explanations for the contrasting poor reality of classroom use (Lagrange et al, 2003).

THE ABSENT TEACHER

A criticism of educational policy and discourse on ICT is that the predominant focus has been on technology rather than education (Selwyn, 1999). The picture painted by Lagrange et al (2003) of research on ICT within mathematics education is of a field dominated by “publications about innovative use or new tools and applications” where issues of the integration of technology into ordinary classrooms have been largely neglected. In particular, the voice and role of the teacher has been notably absent. DGS is no exception: in his review of research on dynamic geometry software, Jones (2002) suggests that future research could usefully focus on teacher input and its impact, amongst other issues. Although research has begun to examine the role of the teacher in DGS integration, the practices of ordinary teachers in ordinary classrooms remains an area requiring further investigation (Lagrange 2008).

This study was designed with these issues in mind. The instrumental approach, described in the next section, was used to analyse teacher/pupil interactions in order to elicit teaching techniques which might facilitate pupils’ instrumental genesis.

THEORETICAL BACKGROUND

Instrumental genesis is described as the process by which an artefact is transformed into an instrument by the subject or user (Guin and Trouche, 1999). An artefact is a material or abstract object, given to a subject. An instrument is a psychological construct built from the artefact by the subject internalising its constraints, resources and procedures (Guin and Trouche, 1999). Once the user has achieved instrumentalisation, he is able to reinterpret or reflect on the activity he is engaged in. Drijvers and Gravemeijer (2005) describe instrumental genesis as the “emergence and evolution of utilisation schemes”. A utilisation scheme is a “stable mental organisation” including both technical skills and supporting concepts as a method of using the artefact for a given class of tasks (Drijvers and Gravemeijer, 2005). The interrelation between machine techniques and concepts seems important since Drijvers and Gravemeijer (2005) found that the apparent technical difficulties that students had often had a conceptual background.

The instrumental approach has been mainly developed and applied within the context of computer algebra software (Drijvers and Gravemeijer, 2005) and there remains a question over how general its applicability is. Drijvers and Gravemeijer (2005) cite two examples where the instrumental approach has been applied to DGS. Thus it seems instrumental genesis may be an appropriate tool to analyse observations of student behaviour within a dynamic geometry environment.

RESEARCH CONTEXT AND METHODOLOGY

This study was conducted as part of a Best Practice Research Scholarship-funded project on using DGS as a resource for teaching geometrical proof. Much of the previous research on DGS has focused on pupils in upper secondary school (Jones, 2000). It has been suggested that more research is needed on the impact of dynamic geometry software on students in lower secondary school (Marrades and Gutiérrez, 2000). The decision to conduct the research with the researcher's year 8 class was partly influenced by this consideration. Since the pupils were in year 8, there was an added advantage that they were not subject to public examinations, the curriculum is less pressurised and therefore ethical considerations about deviating from schemes of work were somewhat reduced. The school in which the research was conducted is a private day school for girls. The research was conducted with the highest attaining set in year 8, containing 23 pupils, with girls expected to achieve levels 7 or 8 at Key Stage 3 [1]. In common with several other research studies, this was seen as an advantage since students judged to be above average in mathematical ability are most likely to be able to engage with proving processes and therefore allow meaningful data collection to take place (Jones, 2000; Marrades and Gutiérrez, 2000).

In this paper, I consider data drawn from a sequence of 5 lessons in which pupils were engaged in investigating a series of construction problems in pairs using Cabri Geometre. The tasks were based upon the Phase 1 and 2 tasks developed by Jones (2000) and were intended to progress in difficulty. Each task consisted of a figure which the pupils were to construct in Cabri so that it remained constant under drag. The methods for constructing a figure were linked and developed from previous problems to encourage the pupils to examine how additional constraints might affect the resultant shape. They were prompted to say what the resultant shape was and, importantly, how did they know? The point of the teaching sequence was to encourage pupils to justify or prove these assertions.

The pupils were asked to choose a construction of their choice and produce a Power-point presentation on why their construction had worked which was presented to the class. Printouts of the pupils' Power-point presentations and audiotape recordings of their presentations to the class form one part of the data collected. During the lessons, the researcher carried an audiotape so that any teacher/pupil interactions would be recorded: these recordings form another part of the data collected. After the lessons, brief field-notes were made on the major events in the lesson.

The initial stage of data analysis concerned the transcription of tape-recordings made during lessons. Using field notes, the tapes were broken down into major events or "episodes" (Bliss et al, 1996). In the sense described by Bliss et al (1996) these episodes had "an internal coherence"; they were complete conversations which allowed the researcher to "interrupt momentarily, for the purpose of analysis, the 'relentless flow of the lesson'". A second stage of analysis involved going through the transcripts and pupils' work making notes, identifying critical incidents that build towards detailed accounts of practices. The final analysis was based on a grounded approach using narrative techniques (Kvale, 1996) which moved back and forth between the theoretical viewpoint developed in the review of literature and the pupils' work and transcribed episodes. Each step in this process eased the transition from emotionally involved participant towards objective observer. Using the concept of instrumental genesis to achieve a "rich and vivid description of events" (Hitchcock and Hughes, 1995), this study hopes to tease out the threads of a tapestry of complex social interactions to see if techniques for promoting mathematical thinking can be discerned in the weave.

ANALYSIS

From the analysis of data, three teaching techniques emerged for facilitating pupils' instrumental genesis in Cabri. Using excerpts from teacher/pupil dialogue, these techniques are described below, where T represents the teacher throughout.

Unravelling functional dependency in DGS

In common with other students, Pupils H and C experienced difficulty with specifying where they wanted objects to intersect when attempting to construct two circles sharing the same radius. They constructed the first circle successfully and correctly placed the centre of the second circle on its edge. The difficulty arose when they tried to adjust the size of the second circle so that its edge would pass through the centre of the first circle, thus ensuring that they would share a radius. The problem was that they made it *look* like the edge of the second circle passed through the centre of the first circle rather than specifying to Cabri that the circle should go "By this point" – as the Cabri pop-up phrase suggests if you hover over the required centre point. Although their Cabri drawing looked successful, when it was subjected to a drag-test, the circles changed size in relation to each other instead of maintaining their pattern:

T: Yeahhh. That's it because you see this computer program will only do exactly what you tell it so if you just make it look like it… sort of, yeah. I'm going to be able to change the shape of your circle so if you tell it, look….

crackle: teacher using the computer to show how the circle can still be messed up. Then creates a new one "by this point" method to show the difference

T: Ok now try and mess it up, you try and mess it up now mess up one of the other circles yeah… ok so…

There follows some unintelligible comments and crackling then…

H: You think a computer's smart but it's not, you can't just sit there and watch it do it for you, you have to know what to do and you have to <u>tell</u> it to do it so it's like a something…. like it's like a lightswitch.

The difficulties that students have in coming to terms with the concept of functional dependency in geometry exemplifies Drijvers and Gravemeijer's (2005) conception of utilisation schemes in which the technical and conceptual elements co-evolve. Pupil H articulates this point very clearly: "you have to know what to do and you have to tell it to do it". Mathematical knowledge is knowing "what to do" and technical knowledge is required in order to tell the computer to do it. The *gap* in H and C's knowledge was an appreciation of the functional dependencies inherent in Cabri: on the one hand, a conceptual gap of the *necessity* of specifying the required geometrical relationship and, on the other hand, a gap in the technical knowledge of *how* to specify the relationship using Cabri. The teacher explains the *need* to specify the geometrical relationship: the "computer program will only do exactly what you tell it". The teacher goes on to illustrate the technical knowledge of *how* to specify the relationship by contrasting the construction 'by eye', which could still be messed-up, to the "by this point" version in which the geometrical relationships remained intact.

Pupil K had similar difficulties to H and C: although she seemed to be clear about how the circle should be positioned, she appeared unaware of the necessity to specify to Cabri that the circle should go "By this point". Again the teacher makes the technical elements explicit:

> Ok. Keep your hand …[K: uhuh] yeah? So if you actually put it on the point and say I want it "by this point" that's how the comp… that's the only bit of IT you're using. [K: But that's…] That's the only knowledge…IT knowledge you've used. And really then you've had to tell it to do that haven't you?

In this case, the teacher is more direct in making the functional dependencies explicit, by guiding the pupil's construction and referring to the software language "by this point". The teacher even describes this technical knowledge of how to specify the relationship as "IT knowledge", unravelling it from the mathematical knowledge of the geometrical relationship. The teacher again refers to the conceptual necessity of specifying the relationship: "you've had to tell it to do that". Drijvers and Gravemeijer (2005) describe instrumental genesis as the "emergence and evolution of utilisation schemes, in which technical and conceptual elements co-evolve". The role of the teacher in supporting instrumental genesis is partly in making the technical and conceptual elements explicit. In the case of dynamic geometry software such as Cabri, the role of the teacher is to unravel the notion of functional dependency by highlighting the *necessity* of specifying the required geometrical relationship and the technical knowledge of *how* to specify the relationship.

Exploiting dynamic variation to highlight geometric invariance

All the figures presented to the pupils for construction were based on the initial construction of a line which was apparently horizontal. Of course, there is no geometrical reason for the line to be horizontal, the figures had been presented in this way purely for neatness and it had not been given a second thought, until the teacher noticed that all students appeared to be constructing *intentionally* horizontal lines. The pupils had discovered that by pressing the "shift" key whilst constructing a line, the line would snap to the horizontal. According to the pupils, a similar feature of "snapping to a grid" occurs in a piece of completely unrelated software, which was how the discovery was made. Pupil K was insistent that the line should be horizontal:

T: Why do you always insist on that being horizontal? Does it matter if it….

The teacher draws attention to the pupil's misconception and, by dragging, attempts to convey that the horizontal constraint is artificial, that it can be broken without disturbing the figure under construction. As the pupils were presenting their work to the class, it became clear that all groups had produced figures with horizontal lines. The teacher again attempted to question this feature of their constructions but this time in a whole class context. Pupil MC was asked to reconstruct her solution to Problem 2 (a perpendicular bisector) without starting from a horizontal line. She did this successfully on an interactive whiteboard so that the whole class could see. She then dragged the figure, directed by the teacher, changing its orientation to show its invariance, including the situation with the initial line being horizontal. The teacher exploits dynamic variation to highlight the geometric invariance of the construction in order to help pupils differentiate between geometrical relationships which were or were not crucial.

A similar situation occurred when a pair of pupils, MC and ML, successfully completed the construction leading to a square (Problem 4). They both excitedly told the teacher that the shape they had produced was a diamond. The teacher dragged their construction so that the base of the shape was horizontal, at which point they both concurred that the shape was a square. Upon dragging it back to the original position, ML in particular returned to her previous statement that it was a diamond. Repeated dragging, more and more slowly to emphasise the continuous 'transformation' of the shape, convinced both students that the shape was, in fact, always a square. Again the teacher's strategy is to demonstrate the potential of the software, by exploiting dynamic variation to demonstrate the invariance of the constructed shape. Recognising the potential of the software and making its affordances explicit to pupils is a key element in supporting instrumental genesis.

Making connections between DGS and pencil-and-paper

Pupil N had constructed a rhombus but, as in the examples in the previous paragraph, had difficulty identifying the shape due to its unfamiliar orientation. The teacher employs dynamic variation to convince pupil N that the shape is indeed a rhombus but then continues the explanation on paper:

N: Is this a rhombus? But a rhombus supposed to be like tilted so…?

Teacher manipulating the diagram on screen

N: Oh so it can be, it can be any way up and it [T: Oh!] would still be a rhombus.

T: Well yeah… [N to another pupil: Well it is a rhombus.] it's like, look, this is a well no that's not. This a rectangle isn't it? Ok, it's still a rectangle. It's still a rectangle. However much I turn it, it's still a rectangle. Yeah, ok?

Diagram of rectangle drawn on paper and then the paper twisted and turned as a demonstration that orientation doesn't alter the shape.

Guin and Trouche (1999) suggest that teachers should highlight the constraints and limitations of the software to students in order to support their instrumental genesis. In these cases, the teacher is in fact using the dynamic nature of the Cabri software to highlight the constraints and limitations of the paper-and-pencil environment, exposing a misconception and thereby supporting the pupils' instrumental genesis in the more traditional medium. In the case of the tilted rhombus, the teacher sketched a rectangle on paper in order to further illustrate the concept that orientation does not affect the nature of the shape. This sketch was done on paper at the time mainly because it was quicker than constructing the shape on Cabri. The teacher's return to the paper-and-pencil environment is important because it makes a connection between the two environments: although dynamic variation makes it easier to appreciate that orientation does not affect the shape, the concept still holds in a paper-and-pencil environment. The return to paper-and-pencil is thus an attempt by the teacher to "build connections with the official mathematics outside the microworld", a responsibility which Guin and Trouche (1999) identify as being a crucial part of the teacher's role.

DISCUSSION

From the sequence of lessons, three teaching techniques have been distilled that serve to facilitate pupils' instrumental genesis in a DGS context. These techniques are clearly not exhaustive: exploiting anomalies of measurement in Cabri such as rounding errors might be another way to promote mathematical thinking, for example. These techniques are specific to DGS in general and Cabri Geometre. They are also analogous to teaching techniques used in other contexts. Guin and Trouche (1999) suggest that teachers should highlight the constraints and limitations of the software to students: in the case of Derive, the discrete and finite nature of the software. Similarly, a dynamic geometry environment such as Cabri is only a discrete model of Euclidean geometry, despite its continuous appearance. All tools and resources have constraints and limitations. In the case of paper and pencil, a limitation is the static nature of the environment. Thus techniques such as those identified in this paper may apply to any teaching resource. In a sense, the teaching techniques mentioned here essentially highlight general principles of mathematics teaching applied to a specific context, in this case DGS. The resource provides a context for learning but cannot teach. The focus of research needs to shift away from the context, towards teachers and the teaching techniques they may employ in order to aid pupils' instrumental genesis. In this way research on ICT may avoid the criticism that the predominant focus has been on technology rather than education.

NOTES

1. Key Stage 3 covers the first three years of secondary schooling in England: Year 7 (age 11-12), Year 8 (age 12-13) and Year 9 (age 13-14). Average attainment at the end of KS3 is at level 5/6. Level 8 is the highest level possible in maths at KS3.

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