# Symbolic Calculators in Mathematics Lessons – The Case of Calculus

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Received: 24 August 2009 Revised: 7 December 2009

*Many empirical investigations concerning the use of computer algebra systems (CAS) and symbolic calculators (SC) are restricted to a period of only a few weeks. They do not show long-term effects on students understanding. Therefore, a long term project (2003–2012) was started to test the use of symbolic calculators in Bavarian "Gymnasien" (grammar schools) in Germany. The project started with an evaluation in grade 10 (see Weigand, 2008). During the 2006/07 and the following school years the project was implemented in the 11th grade. The content taught was calculus: basic properties of functions, limits, continuity, derivatives, and applications of calculus. The evaluation of the project was intended to give answers to the following questions: How did basic mathematical skills (algebraic transformations, solving equations) change? How did students use the SC? How did they evaluate the use of the new tool? The results show that it takes a long time (one year) to get students acquainted with the technology. Moreover, hopes have not been fulfilled that students of these project classes would improve to a greater degree (especially in abilities which might typically be supported by the use of the SC, e. g. interpreting graphs) than students in the control classes. In addition, a relationship between the mathematical abilities of students and their ability in using the SC in problem-solving processes was made. The results of this study raise questions concerning the adequate way to support and foster the - theoretically quite often emphasised - new possibilities of CAS and SC in the class room.*

## 1 Empirical Research about Symbolic Calculators

In the past, many empirical investigations concerning the use of CAS or symbolic calculators (with CAS) in mathematics teaching have been published (see Guin, Ruthven and Trouche 2005 or Drijvers 2003, Pierce and Stacey 2004. Schneider 2000). However, investigations in this area are often restricted to short-term applications of the computer for a few weeks only and do not show the long-term effects on students' knowledge and abilities.

Recently, some studies have started to cover longer periods. The *e-CoLab* project in France (Aldon, Artigue, Bardini, Baroux-Raymond, Bonnafet, Combes, Guichard, Hérault, Nowak, Salles, Trouche, Xavier and Zuchi 2008) was done in 10th grade. e-CoLab = Expérimentation Collaborative de Laboratoires mathématiques. See:
http://educmath.inrp.fr/Educmath/recherche/equipes-associees/partenariat-inrp/archives/partenariat-inrp-08-09/e-colab/

It showed (once again) how the SC was integrated or - better - how the teacher constructed the "orchestration of the instrumental genesis" (Trouche, 2005) into classroom lessons as the most important factor in making students profit from the SC usage. The one year *TI-Nspire project* of the University of Chichester in secondary mathematics classrooms (Clark-Wilson 2008) showed students being much more involved in exploratory mathematical enquiries, and self-acting activities compared to traditional courses. In particular, the students' understanding of graphs, the relation between algebra and geometry and about variables and functions increased significantly.

The *RITEMATHS* projects in Australia integrated the SC in grade 9 and 10. RITEMATHS project is about the use of real problems (R) and information technology (IT) to enhance (E) students' engagement with, and achievement in, mathematics (MATHS).

http://extranet.edfac.unimelb.edu.au/DSME/RITEMATHS/

In particular, it showed the limitations of using SC in these classes. Although the teachers have been very keen to use CAS at the beginning, they eventually used it very little, because the students' effective use turned out to be very hard.

The German *CALIMERO* project (Ingelmann and Bruder, 2007) started with the use of computer algebra 2005 in grade 7 and continued on to the following grades in the next years. CALIMERO = Computeralgebra in Mathematics Lessons: Discovering, Calculating, Organising (translated title). The project showed - compared to control classes - significantly better results of the project classes in heuristic-experimental working in problem-solving situations, but also that the students had issues whilst solving standard problems and working with different representations.

These long-term projects are still running and we may expect more detailed results in the near future.

## 2 Background: The "M³-Project" in the Previous School Years in Grade 10

In the school years 2004, 2005 and 2006 the M[3]-project started with the use of symbolic calculators (SC) in grade 10. M[3]: **M**odel-project New **M**edia in **M**athematics lessons

The students used the TI-Voyage 200. The topics taught were power functions, exponential and logarithmic functions, measurements of circles, trigonometry, volume and surface area of cylinders, cones and spheres. The students were allowed to use the SC in mathematics lessons, for homework and in examinations. The results of these project classes have been compared in a pre-post-test design with control classes, in which the students have had traditional lectures without any use of graphing calculators or SC.

The main results of this project in grade 10 have been (see Weigand 2008):

- The students improved their competence in areas in which a SC can be used beneficially: working with graphs of functions and switching between representations.
- No difference was detected when working with variables, terms and tables. This shows in particular that algebra skills did not remain underdeveloped with the SC classes.
- The frequently made claim of a "scissors effect" concerning the use of new technologies did not occur as there was no development of good students getting better and poor students achieving poorer results. On the contrary, there was an improved performance particularly amongst the poor and average groups, whilst the "good" students improved only slightly. The hypothesis for this result is that the best students were under-challenged and not motivated by being forced to do the paper-and-pencil test without using the SC.
- The mathematical beliefs and attitudes of the students with regard to new tools are ambivalent. One group used the SC very willingly and would like to continue using it in following years. The other group did not enjoy working with the computer, worked with the computer less outside of the course, and had difficulties with its use.
- The questions given in class tests are not significantly different from traditional class tests. The review of experts of given examination questions showed that - compared to traditional examinations - new solution strategies (graphical, numerical solutions, experimental methods) are possible. The computer can be used as a heuristic tool (especially in drawing the graphs of functions by only pressing a button) and as a control instrument, in order to check results obtained with pencil and paper.
- Teachers' lesson reports did at least establish the hypothesis that the calculator is a catalyst for "new" teaching methods. Individual work, as well as partner and group work in mathematics lessons, appeared to be reinforced.

## 3 THE MODEL-PROJECT IN GRADE 11

### 3.1 The Organisation of the Project

During the 2006/07 school year, the project was implemented in grade 11. A total of 732 students at 10 Bavarian grammar schools ("*Gymnasien*") took part in this project. 412 students in 16 classes acted as the "project classes", working with Voyage 200 or TI-Nspire. Schools could apply to participate in the project. The project schools were chosen by the Bavarian Ministry. They are spread out all over the state of Bavaria. The teachers in the project classes mainly did volunteer, but some have also been announced by the heads of their schools. In addition, 320 participants from 11 classes - from the same schools as the project classes - formed control classes for the purposes of quantitative statistical investigation. The students had different previous experiences; some students had been exposed to the SC in the previous grade 10, but other students had come into contact with these systems for the first time in grade 11 (see also Weigand and Bichler 2009). Concerning the practicability of the empirical investigation we had to accept some restrictions. The project schools and classes have not been randomised, the control classes are also not randomized, they are from the same school than the project classes and the teachers of the control classes are asked by the project teachers to participate in the study. Of course we would have wished to have a statistically proper basis of the investigation, but the limited resources - in manpower and finance - and the restriction of the Bavarian Ministry - disturbing the regular work at the schools as less as possible - did not allow this. We are aware that this restricts the validity of the results and that we have to be careful concerning consequences taken out of the study. We - the project team - accepted these restrictions because we wanted to take the unique chance to monitor the development of the integration of the SC into the class room over a period of some years.

The project was funded and supported by the Bavarian Ministry of Education. Texas Instruments provided or supported the symbolic calculators (Voyage 200 and TI-Nspire) for all project classes. The project manager Ewald Bichler is a teacher at the Hans-Leinberger-Gymnasium in Landshut. Ewald. Bichler is very experienced in working with new technologies in schools. He was in permanent contact to the project teachers, ran a learning and teaching platform, made suggestions for unit lessons with the SC and organised, managed and was responsible for the teachers' training. One sort of learning unit which was developed during the project is called "Minute Made Math", more information on:

www.minute-made-math.com

The chair for mathematics education at the University of Wuerzburg (Hans-Georg Weigand) was asked to carry out the scientific evaluation of the project.

The project was mainly taught by teachers with little experience in using SC or CAS. The project teachers held two three-day meetings during which examples of possibilities and opportunities for SC-use were discussed. The teachers jointly prepared a number of suggestions for a range of teaching units intended to highlight the possibilities of using SCs. During the year, the teachers were offered additional learning units by the coordinator (Ewald Bichler). However, there was no uniform overall concept according to which teaching was to be organised in all classes. The personal experience, attitudes and circumstances at the individual schools were too different for this to be possible.

### 3.2 Contents and Research Questions

In grade 11, calculus is taught all over Germany. The content taught is subdivided into basic properties of functions (symmetry, monotonicity, variations in function terms and their impact on graphs), limits, continuity, differentiability, derivation rules, derivation function(s) and applications of differential calculus ("classical" functions discussion, extreme value problems).

During the meetings with the teachers at the beginning and in the middle of the school year, a theoretical frame of the use of SC in the classroom was discussed. In particular, a short insight into the theory of instrumentation was presented and explained with examples (Artigue, 2002 and Trouche, 2005).

Concerning the integration of SC into the problem-solving process, we identified three time zones using the SC: First, at the beginning of the problem-solving process or a concept formation process (SC as a "discoverer"), second in the middle of the process (SC as "solver") and thirdly at the end of the process (SC as a "controller"). We also emphasized the "rule of three" while working with representations: if possible, a problem or the solution of the problem should be represented on a symbolical, graphical and numerical level.

In the following, a selection of research questions from the project is presented. The first two questions relate to the knowledge and the abilities of the students compared to "control classes". The third question asks for the competencies of students in the SC-usage, and the last two questions deal with attitudes and beliefs of students concerning the SC or rather the evaluation of the teachers concerning the project.

- Can any differences be ascertained in terms of core mathematical abilities (term manipulations, interpretation of graphs, solving equations, working with tables, and working with formulae) between the project and the control classes after one year?
- Can different effects of SC-use be ascertained with "good" and "weak" students? The performance criteria used relate to the results of the pre-tests at the beginning of the school year.
- To what extent have students mastered SC at the end of the year? How do the students work with SC during problem-solving processes?
- Do mathematical attitudes or beliefs develop amongst students using the SC?
- How do teachers of the project classes evaluate their teaching with the SC?

To answer the first and second questions, we chose a (classical) pre- and post-test-design – tests using paper and pencil but no calculator – in project and control classes (see www.dmuw.de/weigand/2010/m3 Appendix 1). To answer the third question, the project classes took a test using SC in the middle (February 2007) and at the end (June 2007) of the school year, in which they were asked to record their working methods with the SC in a questionnaire which they completed immediately *after* the test. This research method gives a self-evaluation of the students concerning their individual use of SC in the problem-solving process.

A questionnaire was developed, with answers given according to a five-point scale and questions allowing open answers, in order to answer the fourth question. To answer the fifth question, the teachers filled out a monthly questionnaire as well as one at the end of the school year.

## 4 Evaluation of Pre- and Post-Tests

### 4.1 Results

The pre- and post-test questions are given in Appendix 1 (www.dmuw.de/weigand/2010/m3). These questions can be divided into the following groups: doing "classical" simplification of terms (questions 1 and 2), solving equations (question 3 and 4), understanding the concept of root functions (question 5), seeing the relationships between graph and term (questions 6 to 8) and interpreting graphs (question 9). The post-test was the same as the pre-test. In the following diagram, the *differences* between the average scores achieved for each question in the pre- and post- tests for the project and the control classes are shown. Therefore, the "average performance increase" is measured for each question.

In questions 5 and 7, the results of the project classes are significantly better than those of the control classes (t-Test: question 5: p-value = 0.01, question 7: p-value = 0.02). However, in questions 6 and 9 they are significantly worse (t-Test: question 6: p-value = 0.01, question 9: p-value = 0.01). Overall, there is no significant difference in the average performance increase between the project and control classes.

The test results can be interpreted in a positive way for the project classes, as there are no differences in terms of classical technical and manual abilities and skills. However, this investigation has deflated hopes that the ability to interpret graphs and to transfer between different forms of representation is automatically improved by the use of the SC.

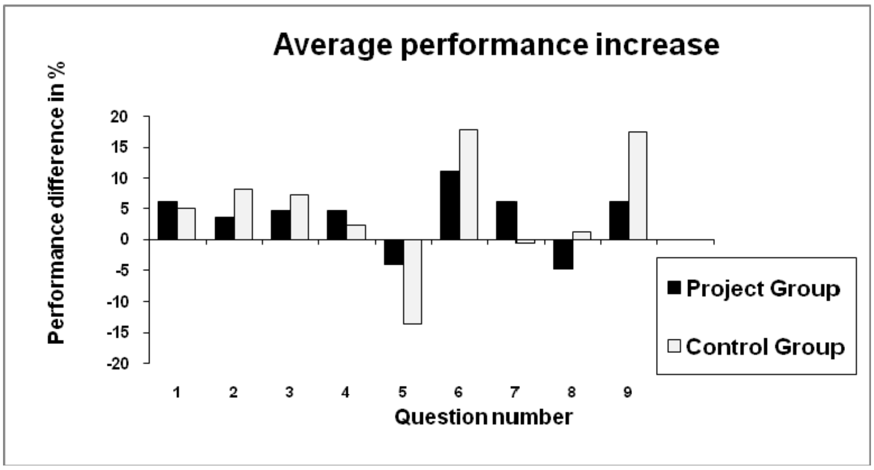


Figure 1 Average performance increase of the project and the control classes in the nine pre- and post-test-questions (PP-Questions)

Comparing the relatively worse result of the project classes to the control classes (especially for questions 6 and 9), there are two possible hypotheses which led to a change in the evaluation and teaching method in the following years.

In accordance with the results of the pre-test, the test participants were divided into categories: "weak", "average" and "good". The "good" students from the upper performance quarter, the "weak" students from the lowest performance quarter, and the "average" students from the two central performance quarters.

The following result was obtained when the performances of these groups were compared in terms of pre- and post tests.

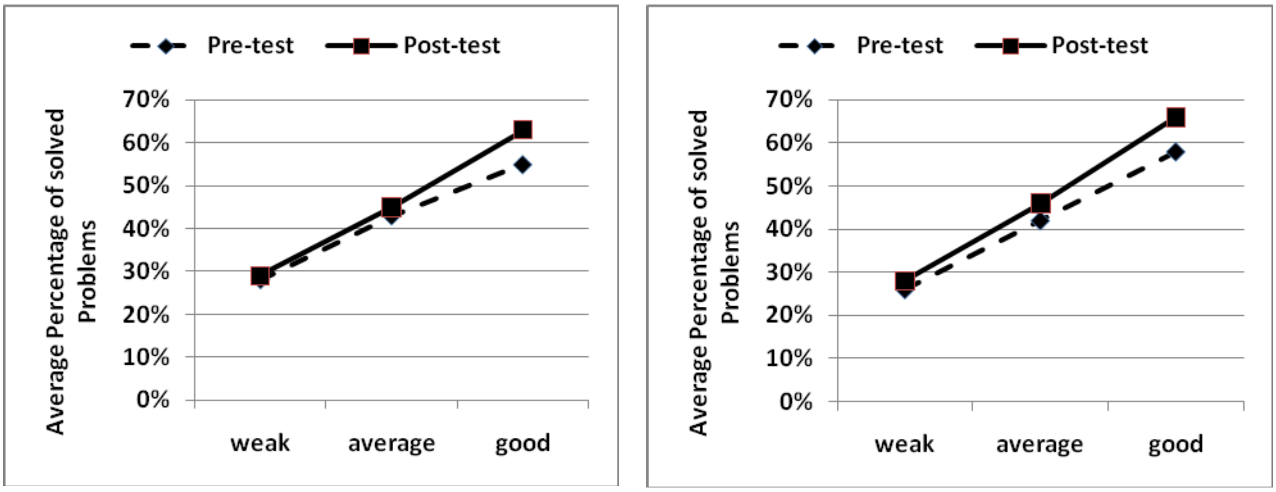


Figure 2 Performances of the project classes (left) and the control classes (right)

Compared with tests carried out in recent years in grade 10 (see Weigand, 2008), different behaviours were demonstrated. Whilst the "weak" students achieved a greater performance increase than the "average" and "good" students in grade 10, the "good" students - both in the control and project classes - improved more markedly (by 8 percentage points) than the "average" and "weak" students (by 3 percentage points and 1-2 percentage points respectively) in the grade 11 test.

The differences between "weak" and "good" groups can be found in the understanding of concepts (question 5) and the transfer between different forms of representation (between graph and equation - questions 8 and 9). Considering the weak students, the lack of performance increase is attributable to the bigger cognitive challenges posed by calculus, which may have taken some students to the limits of their abilities that they were no longer able to follow lessons ("dropout effect").

If we compare the increases in performance for individual tasks between boys and girls, the boys in the project classes improve significantly more than the girls i.e. in questions 2 and 5 (both concern term manipulations). In general however, there were no significant differences between boys and girls.

### 4.1 Interpretations and consequences of the results

The (comparatively) worse results of the project classes could be due to several reasons. Firstly, students in the project classes were no longer adequately challenged or motivated to tackle these types of "traditional" test questions with enthusiasm. With the help of the SC, students had solved much more interesting questions during lessons. The results made us think about the pre-post-test-design. Self-evidently we are aware of our special situation of the test construction and especially of the method of choosing the project- and the control-classes (see Section 3.1). For organisational reasons, we have not been able to build them under statistical objective criteria, choosing them as a random sample. Moreover, our control classes have been at the same schools as the project classes and the students and the teacher of both groups have been in contact. This restricts the validity of the results and makes us very cautious in generalising consequences. But the results of our empirical study have made us think about whether the used pre-post-tests are the correct method to demonstrate the benefits of the working with the SC. If you ask for the benefit of a special new teaching method you may not only ask whether it is "better" or "not better" than another method, you may also ask whether it makes students think and work "differently" or "in another way". You not only assess the *product* but more the *process* of learning and teaching. In our long-standing research project, this led us away from the current quantitative research method and we emphasised more qualitative methods, such as interviews and classroom observations.

Secondly, the poor results of the project classes when determining functional equations from specified graphs (question 6) could be due to the fact that the students in the project classes, compared to the students in the control classes, have seen a large number of graphs during the course of the year. They have therefore probably overtaxed themselves by diversity. However, the students in the control class have worked more often with standard functions like linear, quadratic and sine functions throughout the school year, making them able to handle these functions in the post-test better. We did not want to give up the possibility of creating a great variety of different graphs, because this is a major advantage of the new tools in mathematics education. However we reduced the number of different graphs in one lesson, forced the students to talk about the viewed representations (this fosters concentration on the representation) and tried, as often as possible, to consolidate the knowledge of the graphs of standard functions.

There is another interesting point concerning the results of the pre- and post-tests. If we consider both the average as well as the standard deviation of the performance increases, an interesting fact emerges.

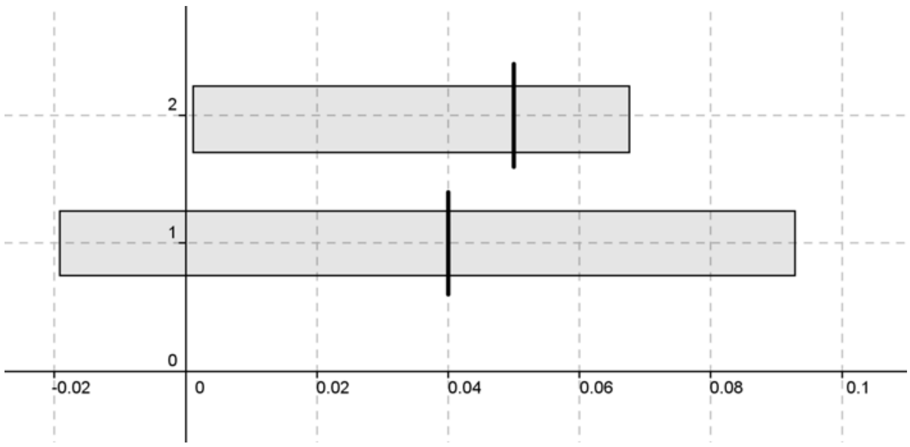



Figure 3 Box plots with the average value and the upper and lower quartile of average performance increases in project (1) and control classes (2)

With an almost identical average value, it becomes apparent that the differences in performance are more varied in the project classes than in the control classes. Therefore, there are students in the project classes who benefit more from SC-use than students in the control classes. However, there are also students whose results deteriorate compared with the initial test. In contrast to the situation in grade 10 (see paragraph 1), the "scissors-effect" - although it is quite small - did occur in grade 11.

## 5. THE SYMBOLIC-CALCULATOR-TESTS (SC-TESTS)

In the middle (February) and at the end (June) of the school year, the project classes took a test where they were allowed to use the SC. The use of the SC was optional for the students, i.e. they decided themselves whether or not they would use the calculator. The two tests consisted out of four questions each (see www.dmuw.de/weigand/2010/m3 - Appendix 2). In order to establish how calculators were used, we applied a new investigation method: the students completed a questionnaire on SC-use immediately *after the test*, giving details of whether and how they used the calculator. This test was intended to answer the following questions:

- How do students use the calculator?
- In which phases of a problem-solving process do the students use the calculator?
- Which functionalities (symbolical, graphical or numerical) do the students use?

This research method gives an insight into the working style of all the students participating in the test. Evidently, it only reproduces the subjective descriptions given by the students. A comparison of the answers of the students and the test results show large conformity and lead us to the conviction that students answered quite seriously.

In addition, the teachers were presented with a questionnaire regarding the questions immediately *before the test.* This was intended to give details of the difficulties which were expected to occur in terms of the questions.

### 5.2 Results

The following diagrams show how many students used the SC during the tests in February and in June, according to their own statements:

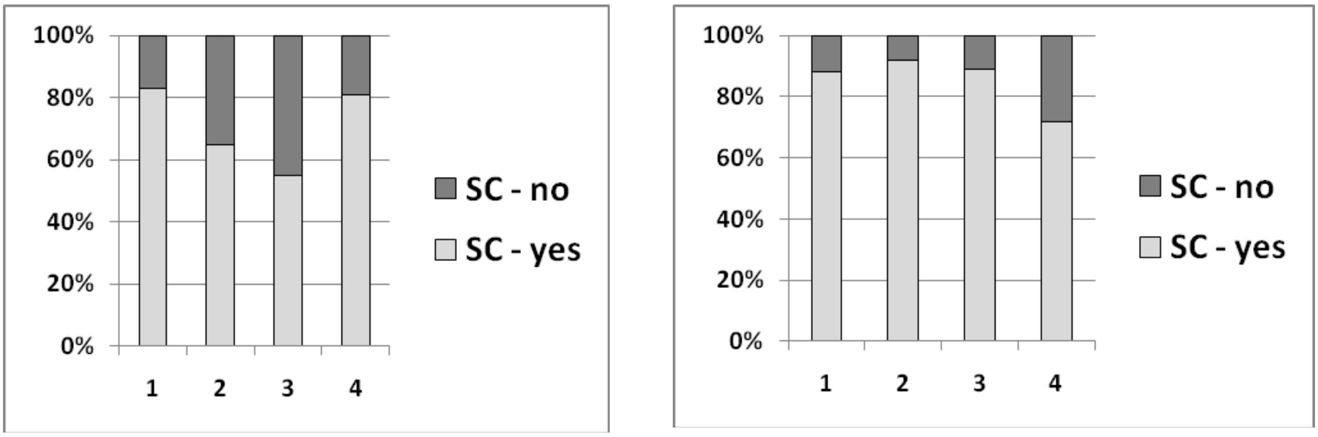
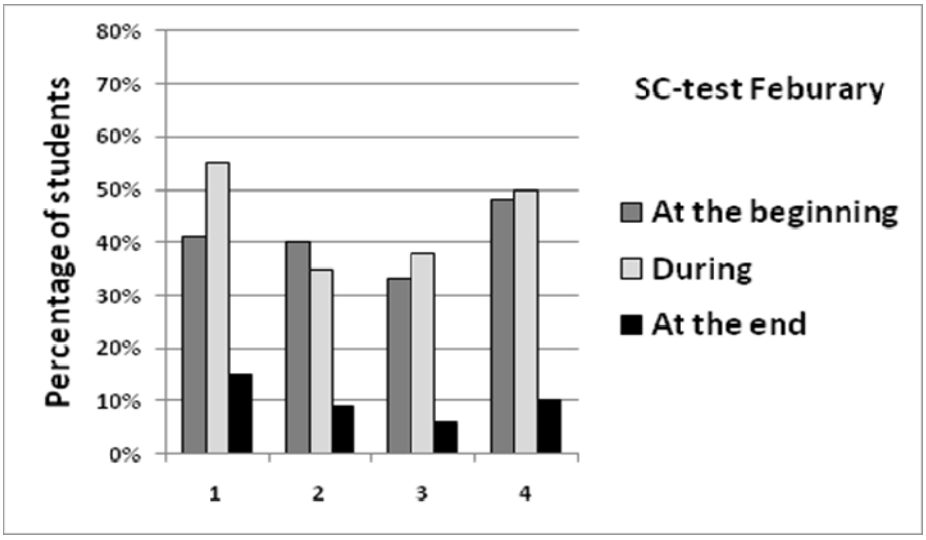



Figure 4 Results of the SC test in February (left) and June (right) 2007

The SC-use in February and in June shows an increase in usage of the calculator. Moreover, those students who used the SC in June when solving the questions scored significantly better than those who did not use it. *We attribute this to the fact that it takes a full school year for students to acquire adequate confidence in the SC.*

The students also provided information in the questionnaire as to whether they used the SC at the beginning, during or at the end of the problem solving process. In Fig 5a and b the percentage of students is given, who used the SC at the beginning, during and at the end of the problem solving process.



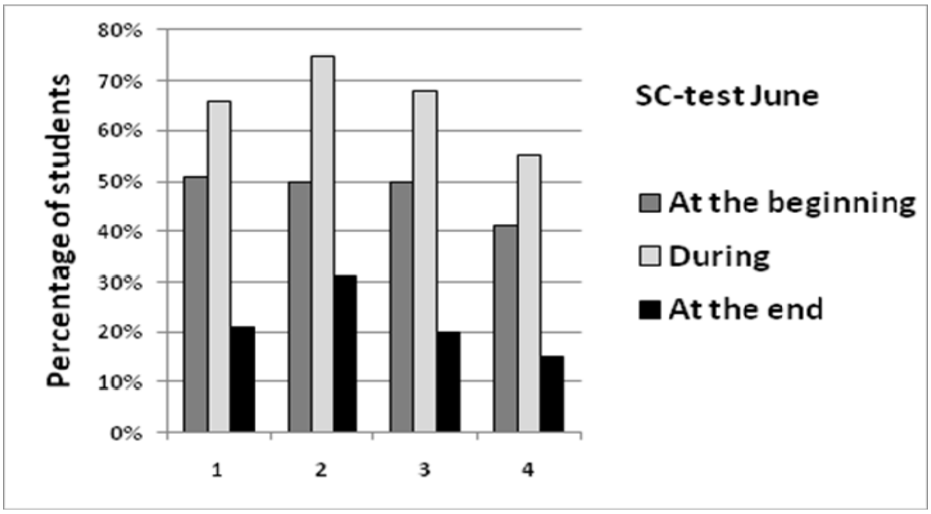




Figure 5a and 5b: Use of the SC during the course of the problem-solving process (according to statements made by the students themselves)

When students integrate the SC into their problem-solving process, it is predominantly used at the beginning and during the problem-solving process. Comparing the middle of the school year and the end, a clear increase in the frequency of positive responses to "during" can be observed. This demonstrates that the SC is more strongly integrated into the problem-solving process by the students at the end of the school year. A slight increase can also be observed "at the end", which displays that the use of checking the solution is gaining importance.

We also asked the students which features they used while working with the SC. To consider the different working styles of the students, the results for each problem have to be analysed separately. Moreover, it is necessary to look at the tests in February and in June separately, because the concept of derivative was only asked in the June test, while the February test explicitly asked for limits. Rather than going too deeply into details, a summary of the results concerning all problems is given. If we summarize the working styles for all problems, we get a rough overview:

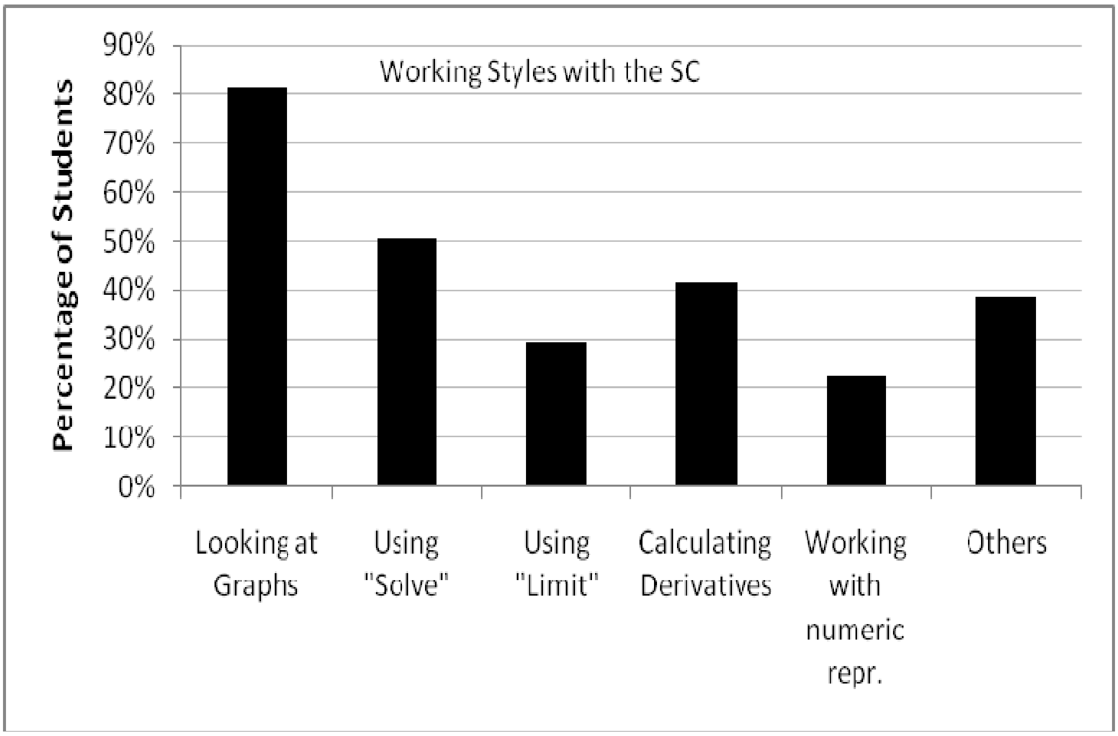



Figure 6 Working styles of students while using the SC

We asked the students which representation they used while solving a problem with the SC. It appears that the students mainly used the symbolical and graphical possibilities of the SC. Numerical use is very limited. Moreover, they are not familiar with the special advantages or disadvantages of these presentations, nor do they use the relationship between the different representations. The type of representation used depends very strongly on the way problems are given to the students. If a "solution of an equation" is asked for, they work mainly on a symbolic level. If an "intersection point of two graphs" is asked for, they work on a graphic level. This shows that the SC is used in a very mechanical way, guided not by the type of problem but by the expressions used in the formulation of the problem. In addition, the type of useage also depends very strongly on the classes and indicates the significance of the teacher and his or her didactic approach. Teachers are different and their teaching methods and beliefs are stable and do only slightly change over a longer time period (see Kendal and Stacey, 2002).

We asked those students who did not use the SC for their reasons. If we again summarise the reasons they gave, we obtain the following overview in Figure 7.

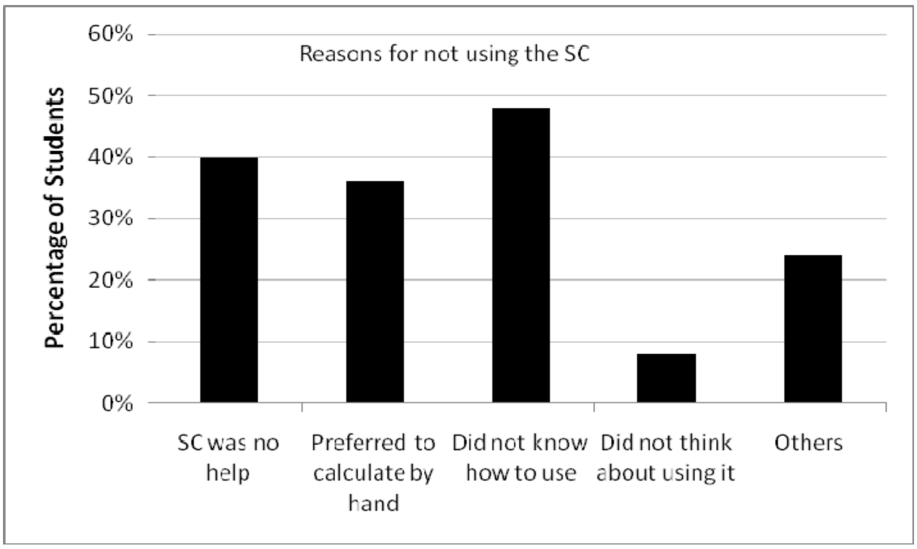
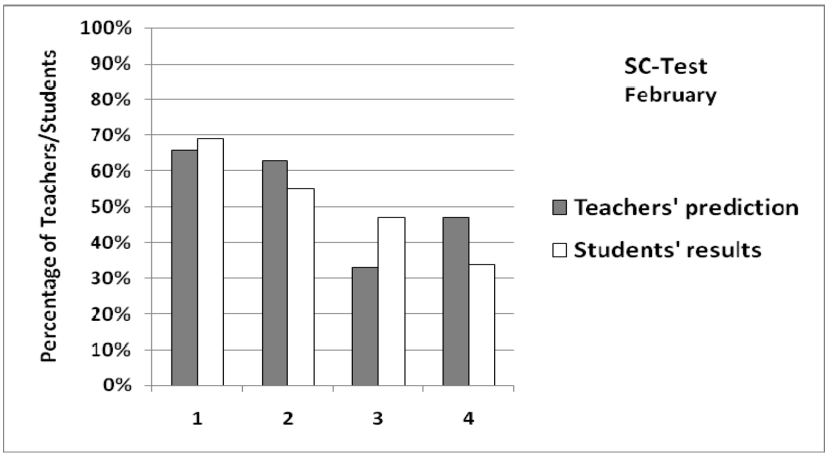



Figure 7 Reasons given by students for not using SC

Not using the calculator relates, regardless of the reasons given, to low results in the SC test. The hypothesis is that the majority of these students have not been able to see the relationship between the mathematical problem and the use of the SC. Thus, it is assumed that the answers "SC was no help" and "Preferred to calculate by hand" do not reflect the real reasons, but this has to be investigated in a more detailed way in the future.

Before each test was carried out, the teachers provided an estimation of the extent to which students would solve the problems. The question was: "For each problem, a student gets 100 % of the marks for a completely right answer. What do you suggest will be the average score of marks your class gets for problem 1 (2, 3, 4)?" The results are shown in Figures 8a and 8b.



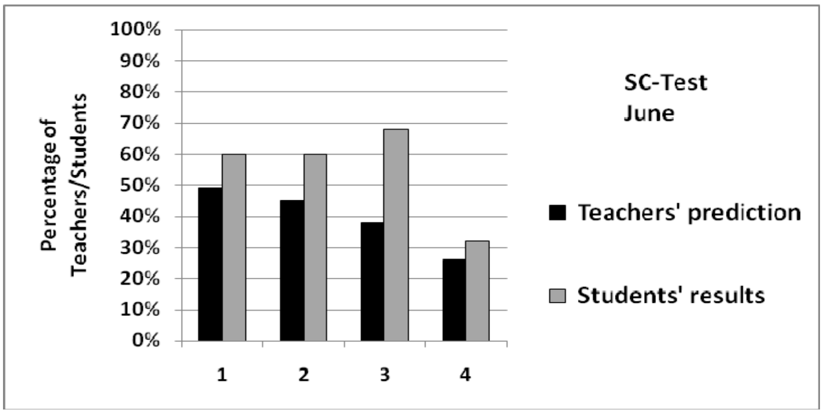




Figure 8a and 8b Comparison of teachers' predicted and students' results in the SC tests

It is noticeable that the teachers underestimated the students performance in the June test. The students improved more then the teachers had predicted. It is also assumed that this is a result of the SC usage for control purposes in the second half of the school year. The teachers totally underestimated this effect.

## 5.2 Evaluation Questions

The questionnaires right after the SC tests also included some evaluation questions about working with the SC. These questions are listed in Table 1 and a comparison of the results is shown in Figure 9.

| | |
|---|---|
| Q1 | Did you find the SC was helpful when completing the tasks? |
| Q2 | Did you experience any difficulties when recording the use of the SC in your solution in written form? |
| Q3 | Did you have any difficulties operating the SC? |
| Q4 | Would you agree with the statement that the SC gave you a feeling of security when completing the tasks? |
| Q5 | When you think of the education you have received using the SC so far, did you find it interesting? |
| Q6 | Have you ever used the SC to solve similar tasks in the past? |

Table 1 The questions used after the SC test

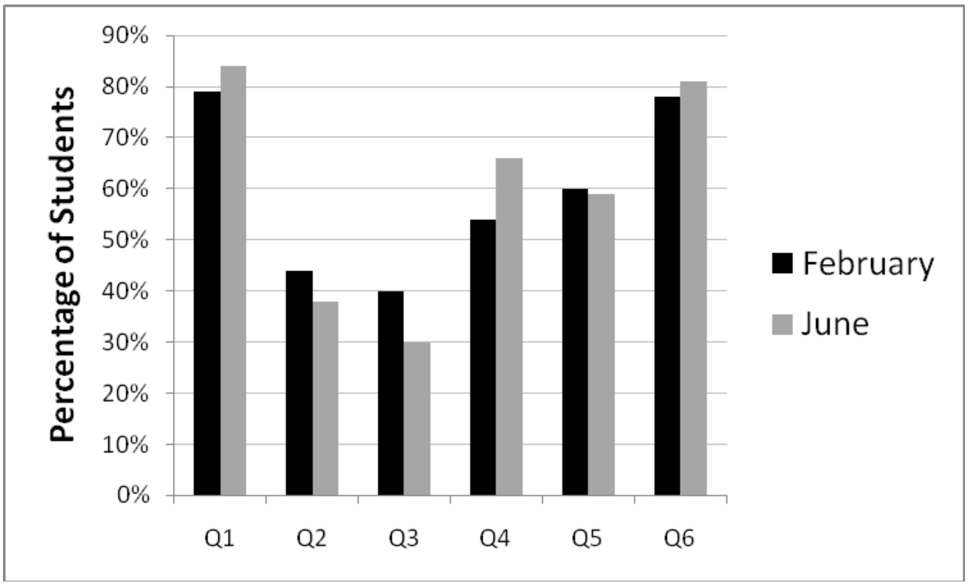



Figure 9 The students' questionnaire. The dark bars represent the responses to the questionnaire after the test in February, the light bars those of the test in June.

Questions Q1 and Q4 show the trend that the SC became more familiar with the SCs towards the end of the school year, as the students find it more helpful and in addition, it gives them confidence when completing tasks. Both, the number of students who had difficulties when documenting the solutions (Q3) and those who reported technical difficulties (Q3) are comparatively high, but the amount altogether decreased compared with the February test.

The responses of the students confirm that familiarity with the new tool requires a very long process of getting used to it. After one year of SC-use, the confidence in and the familiarity with the SC grew. However, there was still a large group of students who experience technical difficulties when operating the SC. Difficulties with the type and manner in which to document the solution decreased, but still remained at a high level. This latter point will continue to be a permanent challenge when working with the SC, as there is no algorithmic solution for the procedure.

## 6 Student and Teacher Questionnaire

To identify how the teachers evaluated their teaching with the SC and how the students' attitudes and beliefs changed during the year of SC usage, a teacher and a student questionnaire has been developed.

### 6.1 The Teacher Questionnaire

Overall, 21 teachers taught in the project classes (16 taught in grade 11, and 5 taught in grade 10). The teachers filled out a monthly online questionnaire and an additional questionnaire at the end of the school year. With the intention of reacting immediately to challenges and difficulties experienced by the students using the SC, the monthly questionnaire was used to evaluate the students' work with the calculator. The questionnaire at the end of the school year was intended to summarise the teachers' experiences over the whole year (see www.dmuw.de/weigand/2010/m3 - Appendix 3).

Rather than giving all the results in detail, the following summary should suffice. The *monthly survey* produced the following results:

- The teachers regularly assessed the students' "feelings" about the SC as being fairly positive.
- 13 % of the teachers use the SC in every lesson and 46 % use it in every other lesson.
- The SC is used above all for function plotting (88 %), for solving equations (73 %) and for term manipulations (65 %). Graphic equation solving takes place relatively rarely (35 %).
- The SC is mainly used for exercising (75 %) and for visualisation (75 %).
- The students worked mainly by themselves (71 %), either alone (56 %) or in groups (50 %).
- Considerable importance is attached to the calculator when checking or controlling the results.
- Teachers see the biggest problem/challenge of the SC usage in the lack of students' knowledge as how to operate the SC.

The questionnaire *at the end of the school year* yielded the following results, presented here, again in summarised form:

- 60 % of the teachers are of the opinion that *content* in their mathematics lessons has not changed compared with traditional lessons. 40 % emphasize the changes of bringing new, interesting examples into the teaching.
- 70 % of the teachers are of the opinion that the *methodology of teaching* has changed: there is

more partner and group work and the students are challenged to present their ideas, suggestions and results.

- All the teachers are of the opinion that students' chances of understanding the content have improved.
- Half the teachers are of the opinion that it is essential that the SC is always available.
- All teachers but one would like to continue to work with the SC.

The SC is regarded by teachers as being a beneficial new tool. It changes teaching methodology, provides an opportunity to integrate new content or at least offers a new form of treatment of old contents. In addition, it serves students as a visualisation and checking instrument. The calculator is generally used in a carefully considered and very restrained way. Moreover, the SC is also used for function plotting, equation solving and term manipulation. The computer algebra component of the SC is regarded as an important element. Numerical and graphical methods appear to play a more subordinate role.

### 6.2 The Student Questionnaire

At the end of the school year a questionnaire with possible answers on a five-point-scale was given to the students of the project classes. (Details of the results are given in www.dmuw.de/weigand/2010/m3 - Appendix 4). The following gives an overview of the results:

- The lessons were regarded by the majority of students as being more interesting, flexible and also easier.
- A large majority sees SCs as being helpful and it provided certainty while solving tasks.
- Approximately a quarter of students state that they had problems with the documentation of solutions in written exams. The students were given the concept of documenting a solution so that may not only have the same lines that the SC-screen shows. It was and still is an on-going problem as to what a documentation should look like. We think that there won't be a general answer to this question.
- A far greater proportion of students (compared to the results in grade 10) would recommend that their fellow students should attend an SC class.
- We see a polarisation into two approximately equal proportions of attitudes reflecting approval and rejection in respect of some questions.

## 7 Summary and Questions for the Future

If we compare the results of this study with the results of empirical short-term investigations – which are listed e. g. in Ellington (2003), who gives a meta-analysis of 54 studies using graphing calculators, Guin et. al. (2005) or Zbiek, Heid and Blume (2007), we can *not* confirm the improvements of students while working with functions and graphs. But we can confirm the realisations of these studies concerning the abilities of the students in mastering term simplifications (like students who did not work with the calculator), the importance of the teacher as a key factor for a successful integration of ICT into mathematics lessons and the difficulties in making the students competent in using the SC in problem solving. If we compare the results with those of other long-term studies, the results of the improvement of mathematical "weak" students confirm the results of the CALIMERO-project (Ingelmann and Bruder, 2007). A greater variety of problem solving strategies is also noticed in the Chichester project (Clark-Wilson, 2008) and we agree with the understanding in the e-Colab project (Aldon et. al., 2008) which deals with the difficulties of teachers and students in looking for new ways of creating new potential for the new tool.

In the following we concentrate on the core results concerning the research questions of this project and we pose questions for up-coming investigations.

- **Methodology of Pre- and Post-Tests**

Hopes have not been fulfilled that students in the project classes would improve to a greater degree in terms of dealing with and interpreting graphs than students in the control classes if we compare the working of the students with traditional paper and pencil problems. However, the students in the project classes have learned to work in a more individualised way, they changed their working style, e. g. working with functions and equations, and they became acquainted with some new examples.

Although we are aware of the special design of our study (see Section 3.1), we raise the question as to whether the pre- and post-test methodology used is an adequate method to answer the questions concerning the new knowledge and the new abilities students in the project classes gained. In the next few years we will be concentrating more on the competences students acquired during the year and we will switch over to qualitative investigations (e. g. interviews with students), to get to know more about the reasons why the students are able to accomplish some tasks but not others?

- **Relation between tool-use and mathematical competencies**

The test with the SC showed that the "weak" students (based on the results of the pre-test) also have had problems with the use of the SC in problem-solving processes. This shows that without basic knowledge students are not able to use the SC in an adequate way. If we take the results in 10$^{th}$ grade showing that especially the weaker students may benefit from the SC-use (Weigand, 2008), we raise the questions, how it is possible to give especially weak students more competencies in the SC-use and how the SC may contribute to an improvement of their mathematical abilities and understanding.

- **Developing tool-competencies**

It takes a long time (in our project between over half a year up to one year) to get the students acquainted with the new tool. In the past we have often underestimated the technical problems of using the SC. The technical abilities have to be trained in a close relation to the contents.

Working with the SC requires different competences. On the one hand, there are different levels of working with the tool – using the SC as a calculator for numerical calculations, as a graph-plotting tool, as a spreadsheet or as a tool for symbolic calculations. On the other hand, there are different demands when using the SC that are dependent on the content. This brought us to the construction of a competence model for using the SC or more generally when working with new technologies (Weigand, 2009). What does a competence model which reveals information about the relation between understanding and tool competencies look like?

- **Improving Mathematical Beliefs concerning the SC-use**

When working with new technologies, polarisation occurs in that some students benefit greatly from SC-use, whereas, for other students, SC-use inhibits performance or even decreases performance levels. Two thirds of students are of the opinion that the SC was helpful and made them more secure and they classify their lessons as "interesting". Approximately one third of students do not share this view. Are there ways to convince all students - or more students - of the benefits of the SC?

- **Improving teaching lessons through classroom materials**

The teachers are very content with the SC in the classroom. Nearly all of them would like to move forwards in this way. They see the chance of changing teaching methods and they are convinced that the students have a better chance to understand. More than half of the teachers don't see a change of the contents in their teaching, whereas others emphasise the possibility of working with new examples.

At the moment, a group of teachers from the $M^3$-project is working on a syllabus for teaching with new technologies in grade 10 and 11. Some more "Minute Made Math" files have been developed. See: www.minute-made-math.com. Will these materials, developed by experienced experts, be accepted by all (many) teachers and will it lead to an improvement of the mathematical and tool competencies of the students?

The project will be continued in the next years as a model project in Bavaria with a greater number of project classes. These project classes will pass their final written mathematics examination with the SC. In 2011 all the Bavarian schools (Gymnasien) are allowed to choose whether they will allow the use of the SC in the final mathematics examinations.

Concerning the research in the next years, the $M^3$-project will focus on three particular aspects:

- Developing, based on the experience of the project in the last years, a lesson concept for grade 10, 11 and 12 to support all teachers of the project classes with materials to be used in class.
- Referring to the three-dimensional PISA-model (OECD 1999 and 2003) with the "dimensions" *content* (numbers, space and shape, change, …), *basic competencies* (communication, argumentation, modelling, …) and *cognitive activation* (reproduction, connections, reflections), a competence model will be developed and evaluated that is based on the three dimensions "Concept Understanding", "Tool Competences" and "Cognitive Activation". For a first draft see Weigand (2009).
- Naturally, final examinations affect teaching in the classroom. Based on the experience of other countries and other states in Germany, models for examination questions have to be developed and tested.

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**BIOGRAPHICAL NOTES**

Hans-Georg Weigand is Professor for Mathematics Education at the Mathematics Department at the University of Wuerzburg (Germany). He has been a Visiting Professor at the University of Illinois (USA) twice. His main research interests are the impact of new technologies in mathematics education, especially in geometry, algebra and calculus. He has created online courses and learning systems for blended learning in mathematics teacher education. He has written many papers in mathematics education as well as textbooks for computers in mathematics education and for the teaching and learning of algebra. He has been the President of the German Society for Mathematics Education (GDM) since April 2007.

Ewald Bichler studied mathematics and physics and has taught for more than 10 years at the Hans-Leinberger-Gymnasium in Landshut (Germnany). He is a long-term member of a working group of Bavarian teachers who evaluated the computer use in mathematics lessons. In the early days of its development, this group developed materials for the integration of computer algebra systems into mathematics classes in Bavaria. For some years, he has been the coordinator of the Bavarian M^3-project (Model-Project-Media- Mathematics), which develops ideas and strategies to integrate handheld technology into mathematics lessons and the associated teacher training. Ewald Bichler will finish his dissertation about the M3-project in 2010.