Teaching with a Symbolic Calculator in 10th Grade – Evaluation of a One Year Project

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A one year project was started in the school year 2003/04 to test the use of symbolic calculators (SC) – the TI-Voyage 200 – in six 10th grade classes of three grammar schools in Bavaria (Germany). The project was repeated in school year 2004/05. The evaluation of the project was intended give answers to the following questions: how basic mathematical skills (algebraic transformations, solving equations, working with tables and formulas) changed; how the questions posed in examinations changed if the students were allowed to use a symbolic calculator (with CAS); how the students evaluated the use of the new tool; and how teaching styles and methods changed in the mathematics lessons. This article presents the results of this project.

1 THE USE OF SYMBOLIC CALCULATORS (SC) IN COMMON MATHEMATICS LESSONS

Even though in many countries symbolic calculators are permitted in schools and examinations (France for instance), they have been – world-wide – only partially integrated into mathematics teaching (in contrast to graphics calculators, which have become mandatory in the majority of countries). The reasons for this are fairly diverse and concern the tool (complex handling, low resolution of the graphics screen, lack of a pedagogical tool), the views of the teachers (insufficient familiarity with the tool, concern about low abilities of the students, importance of traditional mathematics) and also the curricula (inadequate integration of the new tool into the goals of the mathematics lessons). Moreover the complex area of integrating the technology into common school lessons has been underestimated by teachers and researchers (see Trouche 2005a).

In recent times the "theory of instrumental genesis" has been developed to describe the process of integrating calculators into mathematics education. The focus of this theory is to develop learning environments or an "instrumental orchestration", in which the calculator as an *artefact* or *tool* changes into an *instrument*, which is useful for solving problems and which mediates between the user and the mathematical content (see Artigue, 2002, Drijvers and Herwaarden, 2002, Trouche 2005b and Drijvers and Gravemeijer, 2005).

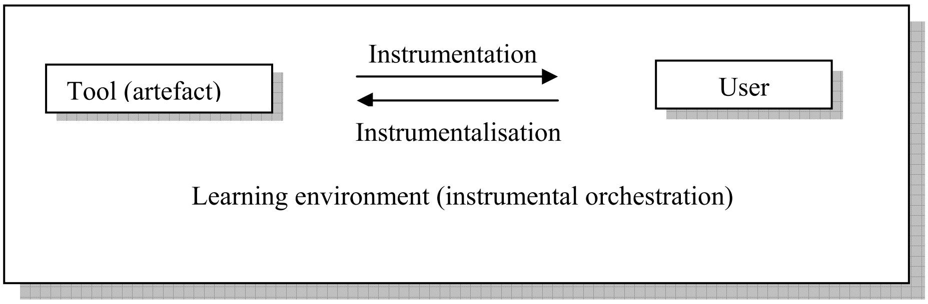

Figure 1 Theory of Instrumental genesis

Through the "process of instrumentation" the user develops mental patterns or models concerning the possibilities and limits of the tool in particular situations (e.g. structure of menus, syntax of commands, and the limits of the internal accuracy of the computer or the screen resolution). By the "process of instrumentalisation" tools are adapted to the needs of the user (through the alteration of menus, the setting up of macros, and programming or defining of commands). This interaction process changes the tool into an *instrument*, which mediates between the user and the mathematical contents. Therein lies the central problem, to develop the learning environment or instrumental orchestration, in which this development of the tool into an instrument will move forward.

In the past, many empirical investigations concerning the use of CAS or symbolic calculators (with CAS) in mathematics teaching have been published (see Lagrange 1999, Weigand and Weller 2001, Peschek and Schneider 2002, Drijvers 2003 and Guin, Ruthven and Trouche, 2005). Prototypical in terms of the aims, implementation and results are surely the Austrian CAS projects. These began with the "Derive Project" at the start of the 1990s, when Austria was the first country in the world to invest in a general licence for a Computer Algebra System for all grammar schools, and through this first experience was able to share a great deal with subsequent users.

The central results of these projects have meanwhile been confirmed by other investigations world wide. The use of a CAS brings a greater meaning to work with diagrams, reinforces experimental work, in which the assumptions were obtained through systematic testing and CAS appears to bring an increase in computer cooperative forms of work. The effects are primarily long term. It is therefore necessary to develop a namely educational concept to evaluate the changes in knowledge and abilities over a longer time period. However, many investigations in this area restrict themselves to the applications of the computer over "just" a few weeks (Schneider, 2000, Barzel and Möller, 2001, Drijvers, 2003, Pierce and Stacey, 2004 and Guin et al, 2005) and do not show the long-term effects on the knowledge and ability of the students.

2 THE TEACHING PROJECT

Six 10th grade classes (referred to in the following as SC-classes) in three Bavarian grammar schools (in total 137 students) used the TI Voyage 200 (referred to here as a symbolic calculator or SC) in lessons and examinations for one year. Four classes belonged to the science section with one additional lesson of "compulsory IT" compared to ordinary classes. In this additional lesson mathematical topics (calculations with π, sequences and series, the Heron algorithm) were dealt with the help of the computer programming language and the spreadsheet program of the SC. Four 10th grade classes (121 students in total, two modern language and two mathematics-science classes) were included as control classes. In the school year 2004/05 the project was repeated with ten 10th grade classes (118 students used the SC and 126 students in the control classes).

There are two specialities, which have to be considered when we evaluate this project. First, the teachers of the 2003/04 classes were familiar with the use of SC and CAS; they had already participated in a working group for the Bavarian government, which created materials for the use of new technologies in mathematics education. These teachers again taught the SC-classes in 2004/05 but there have been also two new "ordinary" teachers. The test results do not show a (significant) difference between the results of the classes of the "experienced" teachers and of the "ordinary" teachers.

The teachers of the SC-classes informed each other about their teaching, but overall they taught their own concept. Second, the students and teachers knew that, due to the guidelines of the Bavarian Ministry of Culture, the SC would not be allowed to be used in the 11th grade. Thus, some pupils might have worked with the tool only on a superficial level, to "pass" the class. In the meantime we continued the project with students who are allowed to use the SC continuously from grade 10 to grade 13. We did not get different results in grade 10 with this population.

The following topics are taught in the 10th grade:

- Calculating with powers and power rules
- Power functions
- Sequences and Series (Different meanings were attached to sequences and series in the different classes. In one of the classes recursively defined sequences and series were dealt with in great detail. In the Bavarian syllabus in contrast only *geometric sequences* given explicitly as mandatory content).
- Exponential and Logarithmic functions
- Measurements of circles
- Trigonometry
- Volume and surface area of cylinders, cones and spheres

The evaluation should deliver some answers to the following questions:

1. What are the changes in central mathematical competences (formation of terms, interpretation of graphs, solving equations, working with tables and formulae) of the SC-classes after one year?

2. Does – as is frequently claimed – the difference between "good" and "poor" students increase with the use of the SC?

3. Do the mathematical attitudes or beliefs develop amongst students using the SC?

4. How do examination questions change as a result of the use of the SC?

5. How do the teachers themselves evaluate their teaching lessons and their change through the use of the SC?

2.1 Test instruments

Questions 1 and 2 were answered by a pre- and post-test design, see the Appendix. All the tests were taken using paper and pencil, the use of SC was not allowed. An external expert assessed the examination questions afterwards in order to answer the third question. A questionnaire was developed, with answers given according to a five point scale and questions allowing open answers, in order to answer the fourth question. To answer the fifth question, the teachers of the SC-classes kept a log of teaching hours and recorded themes of lessons, teaching time using the SC and the method of teaching while using the SC.

We next discuss the answers to the five research questions.

3 RESULTS

3.1 Student Achievements (Question 1)

At the start of the school years 2003/04 and 2004/05 the SC-classes and control classes took a preliminary test and at the end of the school years there was a post-test.

The test results of the total group (SC and control classes) are presented below. Each question was worth a maximum of one point and deductions were carried out in 0.25 increments. So for example in question 1 (simplification of terms), 0.25 of a point was deducted if the binomial formula was correctly used and the formula "except for the *x*" correctly simplified, 0.5 of a point was deducted if the binomial formula was correctly used, but then mistakes appeared in the further simplification and 0.75 of a point was deducted when only the *x* was simplified. All items were marked by the same person.

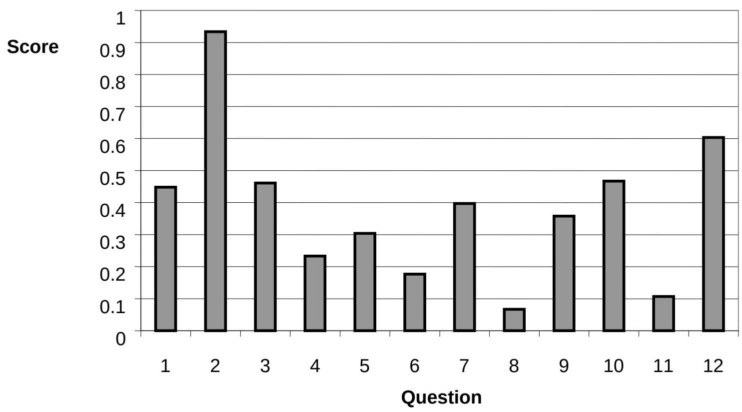

Figure 2 Results of the whole group in the pre-test in school year 2003/04

Figure 2 gives an overview of the degree of difficulty of the questions according to the test group. Question two, which required the recognition of equivalent terms was the best completed. The greatest difficulties were caused by question 8, where a rule for a function should have been developed from a table of values and question 11 which required a quadratic equation to be solved graphically. Figure 3 shows the comparison of the test results for the SC group and the control group in 2003/04.

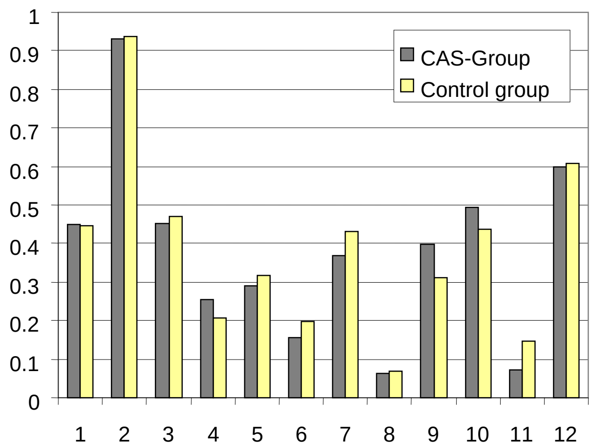

Figure 3 Results of the pre-test for 2003/04

There is no significant difference between the two groups (t = 0.22, df = 128). With regards to individual questions a t-test only showed a significant difference (with 5% significance level) in questions 9 and 11. The SC group dealt more successfully with question 9, whereas the control group was better with question 11. Overall both groups could be regarded as homogeneous. We obtained quite similar results for the school year 2004/05.

At the end of the school years 2003/04 and 2004/05 the post-tests was taken. As in the pre-test, symbolic calculators were not allowed. Questions 4, 6, 8, 10 and 11 were slightly changed – compared to the pre-test – so as to use other functions or rather other function types. Through this the direct comparison between corresponding exercises from the pre-trial and post-trial test was controlled

Figures 4 and 5 show the comparison of the results of the whole group in the pre- and post-test

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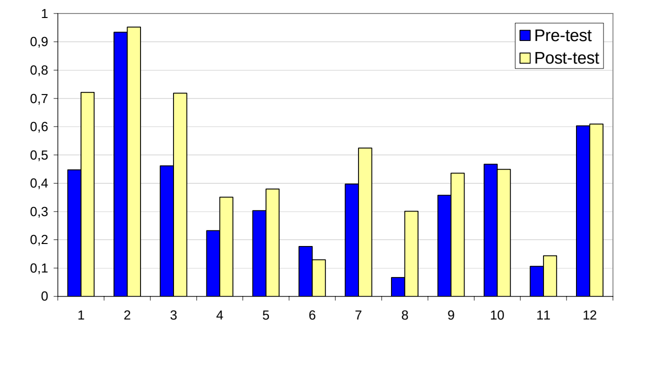

Figure 4 Results of the pre- and post- test for the total group of year 2003/04.

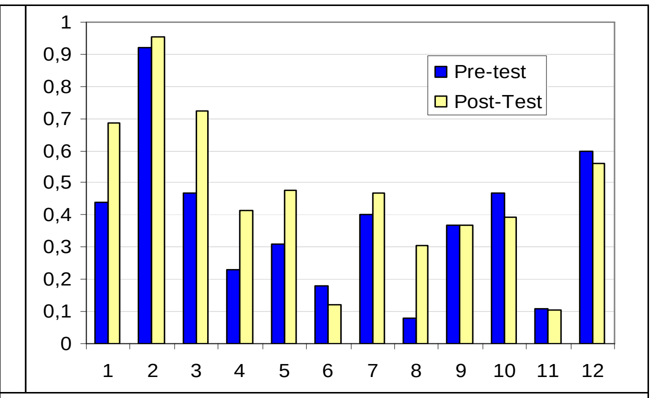

Figure 5 Results of the pre- and post-test for the total group of year 2004/05.

The results in the post-test were generally better than in the pre-test. On the one hand this could have been expected, as the students had progressed in their learning during the year whereas on the other hand to answer the questions very basic principles and knowledge were expected, which was not central to the teaching in the 10th grade.

Questions six and ten were not as well answered in the post-test as the corresponding problems in the pre-test, though this could arguably have been the result of changing the problems too much compared with the questions of the pre-test.

Figures 6 and 7 compare the results from the post-test for the SC and control groups.

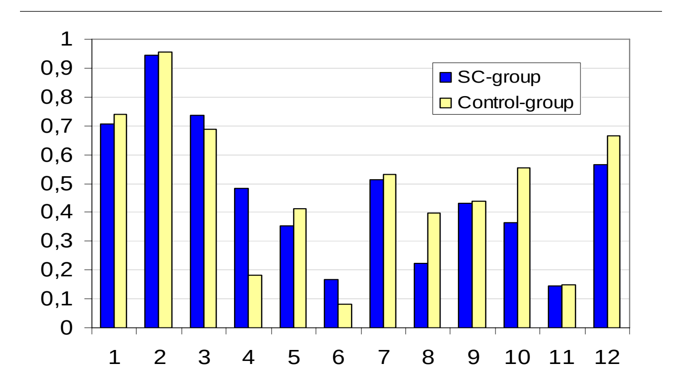

Figure 6 Test results for the SC and control groups of the year 2003/04.

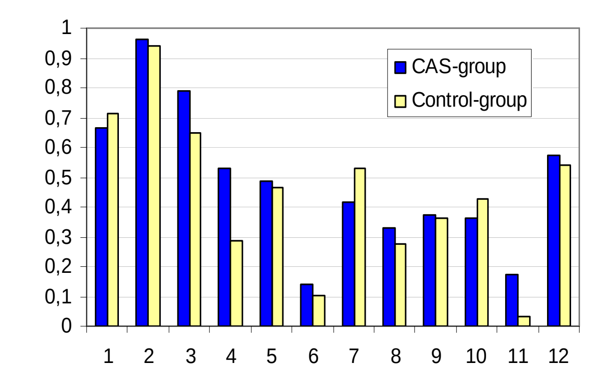

Figure 7 Test results for the SC and control groups of the year 2004/05.

The *t-test* shows that in about half of the questions (1, 2, 3, 5, 7, 9, and 11) no significant differences exist between the SC and control groups. In questions 4 and 6, transfer between the equation of a function and the graph was required. The SC group performed most significantly better in question 4 (t = 4.45, df = 120) than the control group, and question 6 was answered "very significantly" (t = 6.18, df = 120) better by them as well. The control group was significantly more successful in question 8 (t = 3.05, df = 120), "very significantly" better in question 10 (t =4.45, df = 120) as well as significantly better (t = 2.12, df = 120) in question 12. These questions asked for a rule for a function that was presented in a table and required solutions for an equation. Question 12 was a question concerning space geometry. The results for the school year 2004/05 are quite similar. The SC-group answered questions 3 and 4 significantly better.

The results show that the students in the SC-classes also achieved better results when using traditional methods (i.e. pencil and paper) in working with graphs of functions. It should once again be pointed out that the use of the computer in the pre and post trial tests was not allowed. No differences between the SC and control classes were observed in working with variables, terms and tables. This discredits the recurring argument that algebraic skills stay underdeveloped with the use of computers. The results did however show a worsening of the ability of students in the SC-class to solve equations of the type $x^2 + 5x = 0$ or $\sin x = 0.5$. The reasons for this are not obvious from the present data.

3.2 CAS benefits and Student Differences (Question 2)

The test results were additionally used to separate the poorly performing and highly performing groups of students, and potentially to examine differences between the pre- and post-tests. The students were divided into three groups based on the total number of points from the pre-test; poor, average and good. The average score in the pre-test was around 4.5 points. The groups were classified according to the following criteria: poor performing: less than 3.5 points (34 students), average: between 3.5 and 5.5 points (61 students): high performing: more than 5.5 points (30 students).

The performance of these groups was assessed (Figures 8 to 11).

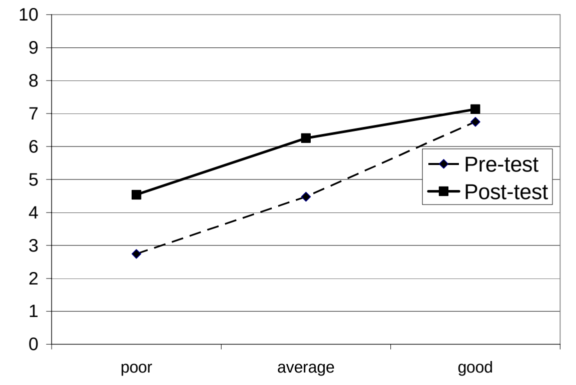

Figure 8 Results of the SC-group for the school year 2003/04.

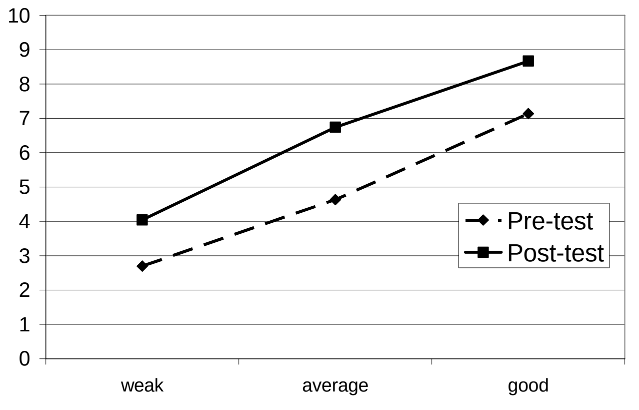

Figure 9 Results of the control group 2003/04.

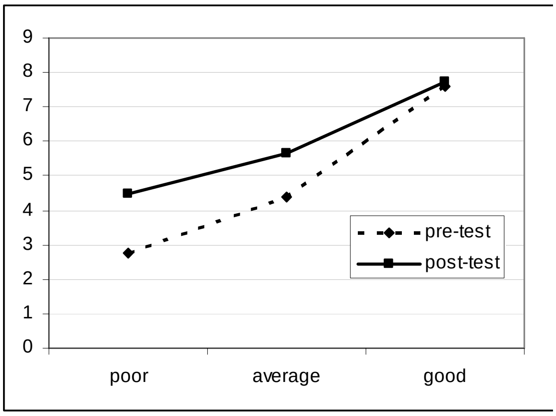

Figure 10 Results of the SC-group for the school year 2004/05.

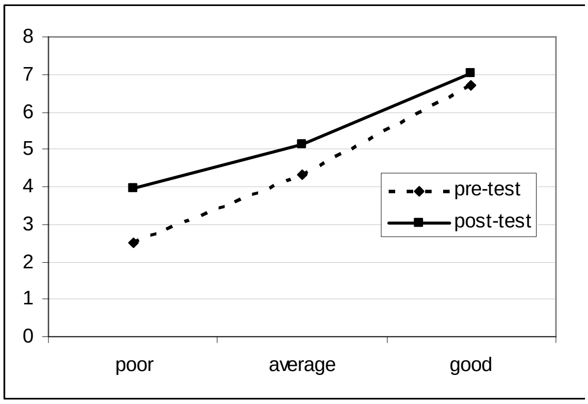

Figure 11 Results of the control group for the school year 2004/05.

It is noticeable that the "scissors effect", namely that good students get better and poor students become even poorer, did not occur here. On the contrary, there was an improved performance particularly amongst the poor and average groups, whilst the "good" students improved only slightly.

The corresponding interpretation of the results of the control classes shows that the improvement in performance across all three groups was virtually uniform, in particular the difference between "poor" and "good" students stayed the same.

3.3 Student Beliefs and Attitudes (Question 3)

The SC-classes filled out a questionnaire evaluating the lessons with SC. The students were required to comment using a five point scale and afterwards answer a further three question which allowed open answers. A total of 131 questionnaires were completed. The distribution of the answers (as percentages) is shown in Table 1. Values are rounded to the nearest whole number. .

++: agree totally, +: agree; o: neither agree nor disagree; –: disagree; – – : totally disagree.

		++	+	O	–	– –
1.	The lessons with the Voyage 200 were more interesting than previous lessons.	16	40	22	11	10
2.	The lessons were easier because of the Voyage 200.	10	36	22	19	13
3.	The lessons were more varied.	21	47	14	9	8
4.	I learnt more in these lessons than in other lessons.	2	15	44	17	22
5.	Mathematics in these lessons gave me more pleasure.	8	24	36	15	17
6.	Using the Voyage 200 has shown me a whole new side to mathematics.	9	35	22	19	15
7.	I participated more actively than in other lessons.	4	11	42	18	25
8.	I used the Voyage 200 outside of lessons and homework.	8	32	11	25	24
9.	I'd really like to carry on using the Voyage 200 in mathematics lessons.	32	21	8	11	27
10.	I would recommend to my friends in the ninth grade going into a class that will use the Voyage 200 without hesitation.	14	31	9	24	21
11.	I used the Voyage 200 frequently for my homework.	17	42	10	19	11
12.	The Voyage 200 is very easy to use.	9	30	9	35	16
13.	I frequently spent a long time searching for particular commands for the Voyage 200.	15	40	5	32	7

Table 1 SC- classes questionnaire responses (all figures are percentages).

The results can be summarised as follows.

- In general the students rated the lessons with the SC to be more interesting (1) and more varied (3). The use of the SC made it possible for the students to discover a new side to mathematics (6).
- Around a half of the students found the lessons easier when using a SC, in comparison to the lessons without SC (2).
- The students were not of the opinion that they learnt more in lessons with the SC (4) or that they participated any more actively (7).
- The students used the computer frequently for their homework (11).
- With regards to the more advanced use of the computer the students divided themselves into two groups: more than a third used the computer outside of the lessons for homework, whereas around a half said they didn't (8). This separation into two groups is also seen with the question of whether the students wished to use the tool further (9, 10) and also with the question relating to difficulties with the uses of the tool (12, 13).

The answers to questions 8 – 13 clearly divide the group into two. One group worked willingly with the calculator, used it outside of the lessons, had no great difficulties in using it and would like to continue using it. The other group did not enjoy working with the computer as much as the former group; used it less outside of the course, and experienced difficulties with its use. According to the teachers, this group always argued (or used this as an excuse) that the computer was not allowed in the next class and therefore there was little point using it to a great extent.

In the following; the most frequent answers to the open questions are presented. Similar answers are grouped together under key words (the number in brackets indicates the frequency with which they were mentioned in the 2003/04 classes.)

Is there something particularly positive that you saw or experienced when using the Voyage 200, in lessons or at home?

Answers:

- The plotting of graphs (17)
- More diverse and drawn from life teaching (16)
- The “Control function“ of the calculation in home-work as well as in school work. (15)
- Simplification of calculations (13)
- Creating tables (11)
- Programming (9)

What caused you the greatest difficulty when using the Voyage 200?

Answers:

- Complicated setup and handling (29)
- Too many commands/programs, difficulties finding the right command /key combinations (26)
- Unclear error messages (11)
- Lengthy/ time consuming input for equations/ terms (9)
- Keys too small (4)
- Unconventional output of results (2)
- English language (2)

What should your teacher change about the lessons with the Voyage 200? Give suggestions!

Answers:

- Conventional methods (for example, carrying out calculations by hand were neglected) (20).
- Work more often with the Voyage 200 (13).
- Suggest more uses for the Voyage 200 (11).
- Additional classes to go through commands for individual topics (7)
- Slower pace or simpler exercises to get used to the handling of the computer (7)

What is particularly striking about these comments is that predominantly mathematics related statements arose in the “positive memories” (plotting graphs, control functions, creating tables). The list of negative memories referred almost exclusively to handling the SC. This shows that many students recognised the possibilities of the tool. Unfortunately the survey was completed anonymously and thus it is impossible to assign the answers to “good” or “weak” students.

3.4 Assessment - The Use of SC in Class Tests (Question 4)

Several different research projects have recently been undertaken in using CAS in written exams (Brown, 2003). The results largely agree that the structure and type of questions should not be fundamentally changed, and that the CAS students will have a greater variety of solution strategies available to them and will therefore be able to choose their own strategy. The analysis of the assessment and the class tests of this project confirm these findings. The vast majority of the questions or problems could be presented in the same way in an examination, in which no computer is allowed. Some of the exercises had already been given by the teacher in previous years.

The questions were deliberately chosen in this way, so that the ability to solve problems by hand with paper and pencil wasn’t left underdeveloped, as students in the 11^{th} grade would no longer be able to use the computer. Moreover for some of questions attempted in class the use of the SC was totally or partially disallowed. In the following the importance of the SC in class tests is analysed. As a result of very limited resources it was unfortunately not possible to analyse all of the student’s solutions.

3.4.1 New Solution Strategies

Both of the following examples show possible extensions of traditional problems, when computers were available.

Example: *A cuboid has length 12cm, width 10cm and height 18cm. The length is shortened to x, the width lengthened to 5x and the height shortened to 3x.*

a) Obtain an expression for the volume V(x) and plot the graph of the function V(x).
b) What is the maximum volume (accurate to the nearest cm^3) that can be achieved by varying the length of the cuboid? For which value of x (accurate to the nearest mm) is this the case?

Exercise a) shows that basic mathematical knowledge is indispensable when working with the computer or the SC. If the user doesn’t give an appropriate region for the plotting, then he/she could obtain highly confusing sections of graph, see Figures 12 and 13.

Question b) is typical of examples requiring an approximate solution with the help of a graph or a table. It shows that the computer allows alternative solutions strategies, as here with the calculation of the extreme values of a third degree function.

3.4.2 Extension of Solution Strategies

Example: *Where does the graph of the function* $f: x \to \frac{1}{2}\cdot(x-12)^4 - 5$ *intercept the axes? Describe the procedure with the computer.*

The answer to this question can be read directly from the graph, obtained by pressing a button (using the “Zero Command”), using a menu command (solve (f(x) = 0, x)) or with the help of a table of values. It could also be solved using paper and pencil. The students therefore decide themselves which method or strategy they will use to solve the problem. This example shows that the use of the SC supplies a larger range of solution strategies.

There are also different solution strategies possible in the following example:

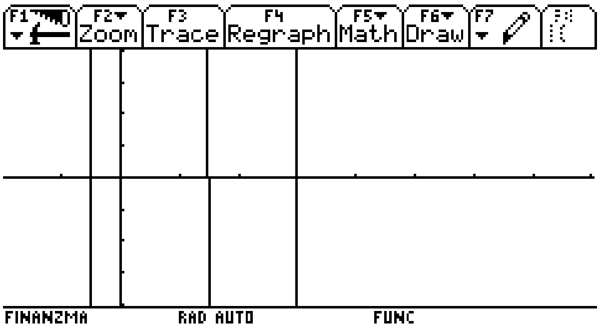

Figure 12 Inappropriate plot area

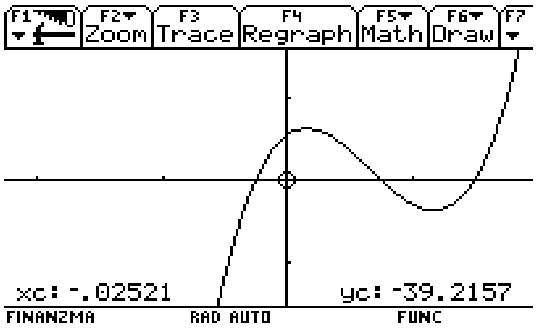

Figure 13 Appropriate plot area

3.4.3 The Computer as a heuristic tool

Example: *Let it be given that for every whole number p the function* $f_p : x \mapsto x^{2p+7}$ *with the respective maximum defined set. What can you state about the symmetry of the graph of* f_p *? (Give a short explanation)*

The calculator can be used to find a solution through the representation of graphs of specific examples, as shown in Figures 14,15 and 16. It also shows that the output in the graph window isn't self explanatory, but rather that the student must have the ability to interpret it.

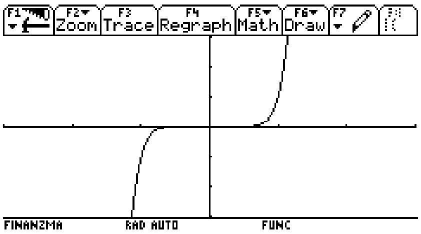

Figure 14 $p = 1$

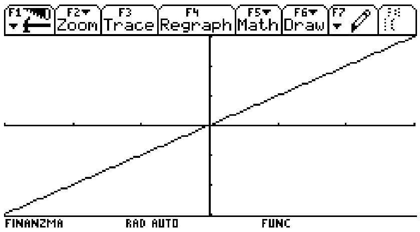

Figure 15 $p = -3$

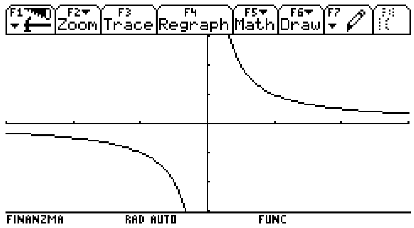

Figure 16 $p = -4$

3.4.4 The Computer as a control instrument

In many respects the computer can be used as a control instrument, in order to check results obtained with pencil and paper. The following example shows this in the case of algebraic simplification.

Example: *Use the rules for indices to simplify the following as far as possible (show each step):*

$$\left(\frac{3ab^{-2}\,b^{4}}{4ab^{-2}}\right)^{2p} : \left(\frac{2b^{-4}}{3a^{-2}}\right)^{-3p}$$

The completely simplified term is $\left(\frac{1}{6}\right)^{p}$.

The output obtained from the Voyage 200 is shown in Figure 17.

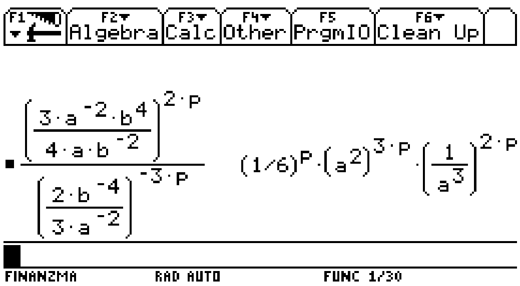

Figure 17 Unexpected algebraic simplification using the Voyage 200

Again it shows that in both the input of terms and the reading of the results from the screen the competence of "structure recognition" is of a central significance.

There was a great deal of optimism in the early years of computer use, one assumed that handling it would soon become very simple. Until now, this goal has not been achieved. The results of the class tests show some mistakes that were directly related to the handling of the calculator. The calculator is a tool or instrument that is not self-explanatory, and its handling must in fact be learnt. Therefore in the examinations of the model classes questions were posed that were directly concerned with the handling of the calculator. For example: "Describe the process with the calculator", "How do you check the results with the TI Voyage 200?" or "Interpret the screen of the TI Voyage 200".

3.5 Self-evaluation of the Teaching Lessons (Question 5)

The teacher of the model classes filled in a record sheet detailing the use of the SC during each lesson of the 2003/04 school year. The topics covered, amount of time spent, method of teaching, and the predominantly used CAS program were recorded each lesson in different teaching phases, see Figure 18.

Content of lesson, exercises, worksheets, etc.	Lesson form while using the SC	Predominantly used SC-window	Length of time in minutes (approx.)…
	☐ Teacher centred ☐ Individual work ☐ Working in pairs ☐ Group work ☐ Project work ☐	☐ Algebra Window ☐ Graphics Window ☐ Table Window ☐ Geometry Window ☐	

Figure 18 Record of lessons

The entries were normally filled in after the lesson by the teacher and therefore offer only an estimate of the timeframe of the use of the SC.

The analysis of the protocols shows that the SC was used in around half of the mathematics lessons. In these lessons, in turn, around half the time was allocated to the use of the SC. That does not mean that the computer was used constantly during this time, but rather that the computer was used in these lessons. This can mean a demonstration of the computer on the part of the teacher or student or the occasional use of the calculator by individual students or also systematic work with the computer by the whole class.

With respect to the teaching form chosen, we found that around 30% of teaching included working in pairs or groups, and likewise 30% individual work or student presentations. On the basis of this data, no valid empirical conclusion is possible; however, considering the records of traditional (i.e. calculator-free) German language mathematics lessons (e. g. the TIMSS-Video-Study 1997, see Hiebert et. al. 1999) does at least establish the hypothesis that the calculator is a catalyst for teaching methods. For this an exact record of the timeframe of the corresponding teaching forms of the experiment and of the control class would be required. This has been consistently claimed for German mathematics lessons, as for instance in the current TIMSS and PISA debates.

The most widely used applications of the palm computer were those of the drawing/graphic (approximately 40%) and the Algebra-(Home), whereas the geometry component was seldom used (around 5%). This affirms the already frequently voiced criticism that the display is far too small for geometric inspection. The remaining share was about 14% for the table application and 10% to other applications, such as the "Program editor". The programming of the computer used for example the Monte-Carlo Method for the calculation of circular area.

The teachers also evaluated their own lessons in two written reports, one half-way through the year and one at the end. Their main points are:

1. The teachers appreciated the advantages of the calculator, with the possibility to easily display functions graphically or in a table. The equations can then be solved graphically or by using the differences between successive values in the table. In particular equations can now be solved, for which up until now only special cases were dealt with, for example polynomial equations of higher degrees, exponential or logarithmic equations. Furthermore there is an access to types of functions which were until now not handled, such as polynomial functions, or discrete logistic growth functions (which could be defined by recursive equations, see Weigand, 2004). Finally, the possibility of using dynamic diagrams are offered, e. g. working with groups of functions or changing table values.

2. The SC is a control instrument for calculations. It allows algebraic transformations or transformations of terms to be checked using individual numeric examples. The experience of the teacher showed that such control skills needed to be developed and that this was frequently only managed by the stronger performing students. Poorly performing students by contrast didn't manage to successfully search for and remove a mistake when there was a discrepancy between the control and original calculations.

3. There are possibilities for *discrete* operations or activities, e. g. working with series and nested intervals. This allows the iterative numerical calculation of powers with irrational exponents and thus at least suggests a way in which the existence of such a number can be demonstrated. Further the calculations of the volumes of cones and spheres are calculated through inscribed cylinders with gradually decreasing heights and iterative calculations with circumferences or circular area.

4. All of the teachers were of the opinion that particularly the poorly performing students were very passive when working with the calculator and were reluctant to familiarise themselves with the use of the tool. They perceived the learning or the use of the computer to be additional "new stuff". For these students the computer remained as a tool or artefact and didn't develop into an instrument.

5. Amongst the students the size and resolution of the display was not seen as troublesome. However the staff felt that the possibility of working with the geometry program was not considered to be geometrically adequate because of the size of the display. Hence the geometry application of the computer was little used on the part of the teacher. It is still an open question as to whether a familiarisation effect would occur with the frequent usage of the tool.

6. The teachers believed that lesson preparation was considerably more time consuming because of the inclusion of a new medium. The delivery of the lesson was also considered to be more challenging particularly as a result of many different problems concerned with both the content and with technical problems.

4 CONCLUSION

The two year-long projects show that the SC was well integrated into the regular teaching of the 10th grade classes. It confirms some findings:

1. The results of the pre- and post-tests confirm a development in competences in areas in which a SC can be used beneficially. Thus the students of the SC-classes achieved a greater improvement – compared to the control class – in working with graphs of functions and switching between representations. Furthermore, no difference was detected working with variables, terms and tables. This shows in particular that algebra skills did not stay underdeveloped with the SC-classes.

2. The improved performance amongst the poor and average students is noticeable. The quite often claimed "scissors effect", that good students get better and poor students become even poorer, did not occur. But it is very precarious that the best quarter of the students of the SC-classes (measured by the result of the pre-test) did not improve (measured by the result of the post-test). The hypothesis for further investigations is, that these students were under challenged by the practised methods of lessons and that up-coming lessons have to support these students individually.

3. The mathematical beliefs and attitudes of the students with regard to new tools are ambivalent. One group used the SC very willingly and would like to continue using it in up-coming years. The other group did not enjoy working with the computer, worked with the computer less outside of the course, and had difficulties with its use. The data of the questionnaire were taken anonymously and don't give correlations to students' achievement in the tests. This could be a research question for further investigations.

4. The questions given in class tests are not significantly different from traditional class tests. The review of experts of the given examination questions shows, that – compared to traditional examinations – new solution strategies (graphical, numerical solutions, experimental methods) are possible, that the computer can be used as a heuristic tool (especially in drawing the graphs of functions by only pressing a button) and as a control instrument, in order to check results obtained with pencil and paper. If you see the many suggestions in the literature for well-constructed new problems in SC-supported classes, more research is necessary to get to know which of these might also be good problems for written examinations. The results of the class tests also show some mistakes that were directly related to the handling of the calculator. The calculator is a tool or instrument that is not self-explanatory, and its use or handling must in fact be learnt.

5. The lesson protocols of the teachers do at least establish the hypothesis that the calculator is a catalyst for "new" teaching methods. Individual work, as well as partner and group work in mathematics lessons_appeared to be reinforced. But teachers claim that particularly the poorly performing students were very passive when working with the calculator. Hence, not only the top group in the class but also the weaker students have to be supported individually.

On account of the positive results of this project, the Bavarian Ministry decided to continue the project. The follow-up project was started in September 2005 with 10 classes using the SC in 10^{th} grade, and they will go on using this new tool over 4 years until their final examination. This allows a systematic investigation of some open questions noticed in the first project, and offers the possibility to evaluate the development of long-term competencies.

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REFERENCES

Artigue, M. (2002) Learning Mathematics in a CAS Environment: The Genesis of a Reflection about Instrumentation and the Dialectics between technical and conceptual Work, *International Journal of Computers for Mathematical Learning*, **7**, 245-274.

Barzel, B. and Möller, R. (2001) About the Use of the TI-92 for an Open Learning Approach to Power Functions – A Teaching Study, *Zentralblatt für Didaktik der Mathematik* (ZDM), **33**(1), 1-5.

Brown, R. (2003) Computer Algebra Systems and Mathematics Examinations: A comparative Study, *International Journal of Computer Algebra in Mathematics Education*, **10**(3), 155-182.

Drijvers, P. and Gravemeijer, K. (2005) Computer algebra as an instrument: examples of algebraic schemes, in: Guin, D., Ruthven, K.and Trouche, L. (eds), The Didactical Challenge of Symbolic Calculators, New York: Springer, 163-196.

Drijvers, P. and Van Herwaarden, O. (2000) Instrumentation of ICT-tools: The case of algebra in a computer algebra environment, *International Journal of Computer Algebra in Mathematics Education* **7**, 255-275.

Drijvers, P. (2003) Learning Algebra in a Computer Algebra Environment, Design Research on the Understanding of the Concept of Parameter, Dissertation, Utrecht.

Guin, D., Ruthven, K.and Trouche L. (eds) (2005) The Didactical Challenge of Symbolic Calculators, New York: Springer.

Lagrange, J.-B. (1999), Complex calculators in the classroom: Theoretical and practical reflections on teaching precalculus, *International Journal of Computers for Mathematical Learning,* 4, 51-81

Peschek, W. and Schneider, E. (2002) Computer algebra systems (CAS) and mathematical communication, *International Journal of Computer Algebra in Mathematics Education*, **9**, 229-242.

Pierce, R. and Stacey, K. (2004) A Framework for Monitoring Progress and Planning Teaching towards the effective Use of Computer Algebra Systems, *International Journal of Computers for Mathematical Learning* **9**, 59-93.

Schneider, E., (2000) Teacher Experiences with the Use of a CAS in a Mathematics Classroom, *International Journal of Computer Algebra in Mathematics Education*, **7**, 119-141.

Hiebert, J., Stigler, J. W. and Manaster, A. B. (1999), Mathematical features of lessons in the TIMSS video study, ZDM, Zentralblatt für Didaktik der Mathematik 31 (6), 196-201

Trouche, L. (2005a) Calculators in Mathematics Education: A rapid Evolution of Tools, with differential Effects, in Guin, D., Ruthven, K. and Trouche L. (eds), The Didactical Challenge of Symbolic Calculators, New York: Springer, 9-39.

Trouche, L. (2005b) Instrumental Genesis, individual and Social Aspects, in Guin, D., Ruthven, K. and Trouche L. (eds), The Didactical Challenge of Symbolic Calculators, New York: Springer, 197-230

Weigand, H.-G. (2004) Sequences - Basic Elements for Discrete Mathematics, *Zentralblatt für Didaktik der Mathematik* **36**(3), 91-97.

Weigand, H.-G., and Weller, H. (2001) Changes of Working Styles in a Computer Algebra Environment - The Case of Functions, *International Journal of Computers for Mathematical Learning*, **6**, 87-111

BIOGRAPHICAL NOTES

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APPENDIX

The pre-test

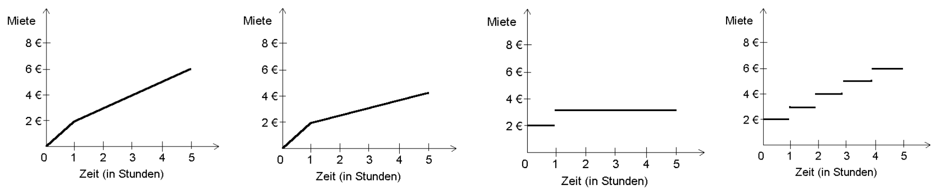
Please note that to save space the questions are not given in their original layout. The graphs are smaller and the space given for calculations and sketching have been omitted.

1. Simplify the following term as far as possible: $\dfrac{x^3 - xy^2}{x^3 - x^2 y} =$

2. Which of these expressions are equal to $2x + y^3$? Circle the appropriate letter (A, B, C or D).

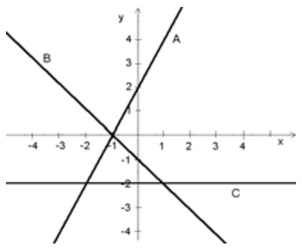
$$A: x^2 + y \cdot y^2 \qquad B: x + x + y^3 \qquad C: x \cdot x + y^3 \qquad D: x + x + y \cdot y^2.$$

3. There are bikes for hire in a park. The first hour (or part thereof) costs 2 € and each further hour 1 €. Which diagram shows this? Circle the corresponding letter A, B, C or D.



A B C D

4. Determine the equations of the three lines A, B and C.

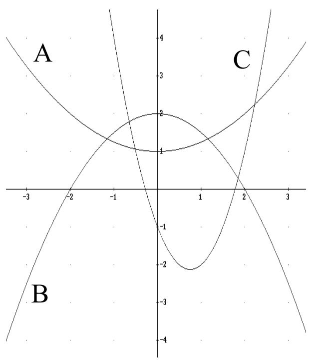


A: ....

B: ....

C: …..............................

5. These equations belong to the corresponding parabolas A, B and C. Which number belongs in each box?



A : $y = \frac{1}{4}x^2 + \square$

B : $y = \square\, x^2 + 2$

C : $y = 2x^2 - 3x + \square$

6. Sketch the graph of the quadratic equation

$y = -0.5(x - 1.5)^2 + 3$.

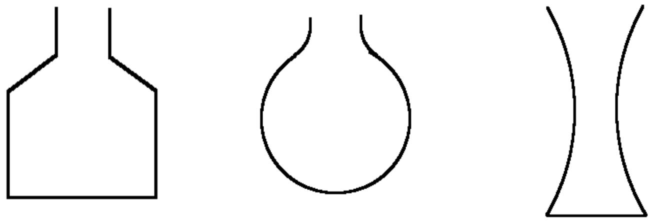
7. The following x and y values are given for a function $x: \mapsto y$. Complete the table:

x	1	2	3	4	5	6
y	-1	0	3	8		

8. Give a rule for a function that fits the following table:

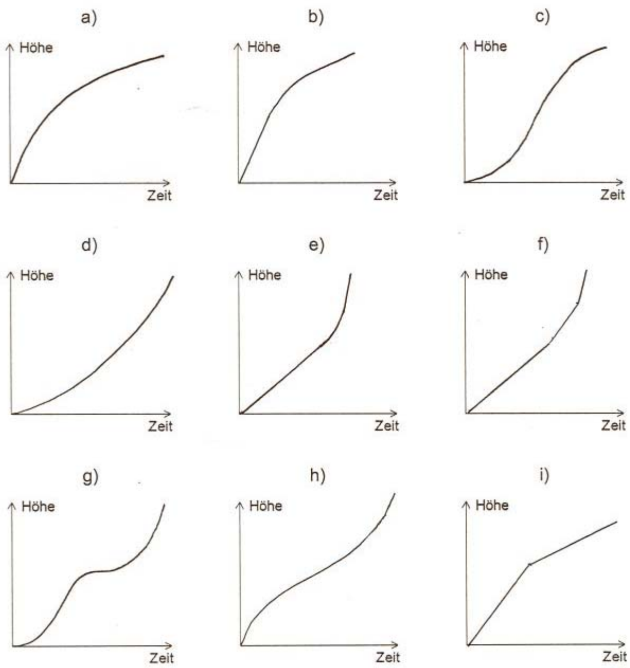
x	-3	-2	-1	0	1	2
y	-2.5	-1	0.5	2	3.5	5

9. Water is poured steadily into the vases shown below. The given graphs show the height of the water as a function of time. Which graph goes with which vase?



Vase A Vase B Vase C

Vase A and graph: ....
Vase B and graph: ....
Vase C and graph: ....

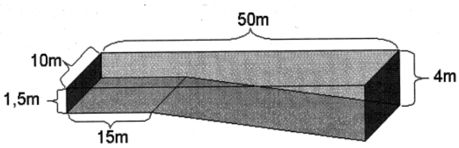


10. Determine the solutions to the following equations in R.

a) $x^2 + 5x = 0$ b) $x^2 = x$.

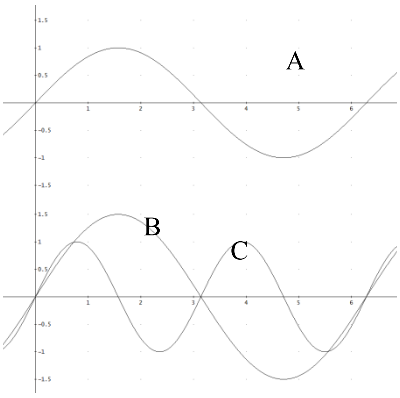
11. **Using a graph** identify approximately the solution to the equation $x^2 - x - 3 = 0$.

12. Calculate the volume of this (completely full) swimming pool.



The post-test: (Only the questions that were changed from the pre trial test are given.)

4. A is the graph of the function $y = \sin(x)$. Determine the equations of the graphs B and C.



A: $y = \sin(x)$

B: ....

C: …...............................

6. Sketch the graphs of the equations

A: $y = \cos(x)$ and B: $y = -2\cos(x) + 0.5$ from –3 to +3.

8. Give a rule for a function that fits the following table:

x	-3	-2	-1	0	1	2
y	$\frac{8}{27}$	$\frac{4}{9}$	$\frac{2}{3}$	1	$\frac{3}{2}$	$\frac{9}{4}$

10. Determine the solution to each of the following equations in R.

a) $x^2 + 5x = 0$ b) $\sin(x) = 0.5$.

11. **Using a graph** identify (approximately) the solution to the equation $\cos(x) = x$.