# CAS WE CAN! – BUT SHOULD WE?

## THE INTEGRATION OF SYMBOLIC CALCULATORS INTO MATHEMATICS LESSONS

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*A long term project (2005–2012) was started to test the use of symbolic calculators (SC) in Bavarian grammar schools (Germany) in grade 10 and 11. The results showed the necessity of having better diagnostic possibilities of the (non)understanding of the students and their ability of working with the SC. How does a student understand a special content? What is the reason for not having mastered a problem: Did she/he not understand the content or has she/he "only" not been able to use the SC in an adequate way? Empirical investigations led to the construction of a model of competence, which reveals information about the relation between understanding, tool competencies and different levels of cognitive activation.*

## 1. BACKGROUND

In the past, many empirical investigations concerning the use of CAS or symbolic calculators (with CAS) in mathematics teaching have been published (see Guin, Ruthven and Trouche 2005). However, many investigations in this area are restricted to applications of the computer for "just" a few weeks and do not show the long-term effects on the knowledge and the ability of students. Only in recent times, there are a few longstanding studies: e-CoLab (France), CASCAT, RITEMATHS (Australia), TI-Nspire and Understanding (UK - Univ. of Chichester), CALIMERO (Germany).

In the school year 2005/06 a long term project (the $M^3$-project) was launched to evaluate the use of symbolic calculators (SC) – the TI-Voyage 200 and the TI-Nspire – in grammar schools (Gymnasien) in Bavaria (Germany). The project was realized in grade 10 and 11. An overview of the empirical investigation and especially of the theoretical background of this project gives Weigand (2008). In the following paragraph – based on the results – a competence model is given, which reveals information about the relation between understanding, SC- or tool competencies and different levels of cognitive activation.

## 2. THE TEACHING PROJECT – GRADE 10 AND 11

### 2.1 The learning contents

In the 10th grade of Bavarian grammar schools, the following contents are taught:

- Calculating with powers and power rules,

- Power functions,
- Exponential and Logarithmic functions,
- Trigonometry.

In grade 11 the main subject is calculus:

- Basic properties of functions (symmetry, monotonicity, variations in function terms and their impact on graphs, …),
- Derivation rules, derivation function(s),
- Applications of differential calculus (“classical” functions discussion, extreme value problems).

## 2.2 Research questions:

At the beginning of this project, we especially concentrated on the following research questions (RQ):

RQ1. Can any differences be ascertained in terms of core mathematical abilities (substitutions, interpretation of graphs, solving equations, working with tables, and working with formulae) between the pilot and the control groups after one year?

RQ2. Can different effects of SC use be ascertained with “good”, “average” and “weak” students?[1]

RQ3. To what extent have students mastered the SC at the end of the year?

RQ4. In which phases of a problem solving activity do the students use the SC?

## 2.3 Test instruments

For the purpose of answering the 1st and 2nd question, we took a (classical) pre- and post-test-design – tests using paper and pencil but no calculator – in pilot and control classes.[2] For the purpose of answering the 3rd and 4th question, the pilot classes took a test using a SC after half of the school year and after one school year. Their working methods with the SC were recorded in a questionnaire which was completed immediately *after* the test. For the results of the pre- and post-tests, see Weigand (2008).

Hopes have not been fulfilled as students in the pilot classes did not improve to a greater degree in terms of dealing with and interpreting graphs than students in the control classes. The hypothesis is that students in the pilot classes have not been adequately challenged or motivated as result of the traditional nature of the test problems. This raises the question whether the used pre- and post-test methodology is an adequate method to answer this question.

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[1] The performance criteria used relate to the results of the pre-tests at the beginning of the school year.

[2] See: www.dmuw.de/weigand/2009/ICTMT9/

## 3. THE SYMBOLIC-CALCULATOR-TESTS (SC-TESTS)

The following diagrams illustrate how many students used the SC during the tests after half a year and after one year– according to their own statements:

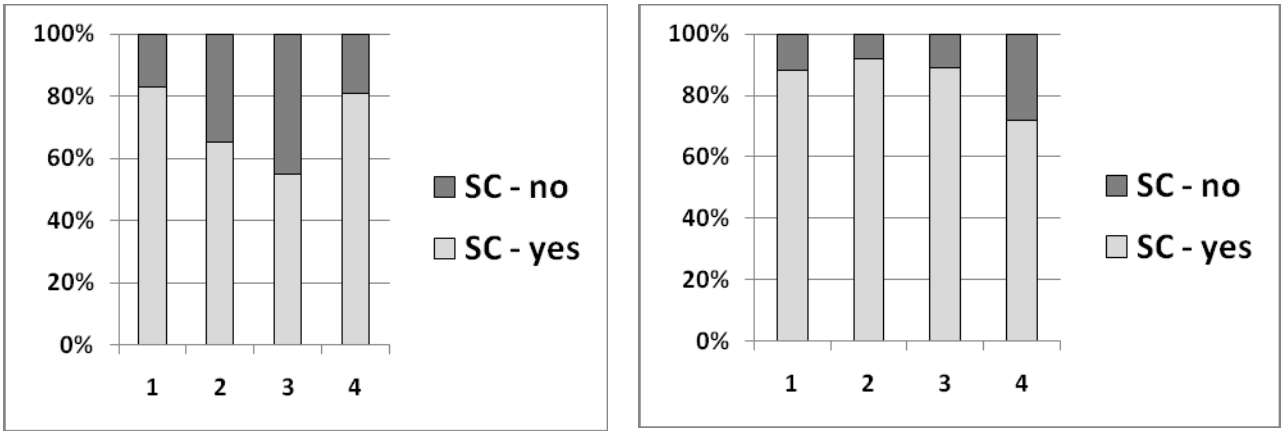



**Figure 4: Results of the SC-test after half a year (left) and on year (right)**

The difference between SC use after half a year and after one year shows an increase in use of the calculator. Moreover, the students who used the SC after one year scored significantly better when solving the questions than the ones who did not use it. We attribute this to the fact that the students need a full school year to acquire adequate confidence in the SC, as well as knowledge of the benefits of its use as a tool when solving problems.

In a questionnaire the students were asked about their difficulties and uncertainties regarding the technical handling of the SC. The responses of the students confirm that familiarity with the new tool requires a very long process of getting used to it. It is surprising that students needed almost a year to establish familiarity with this tool and to use it in an adequate way. After one year of SC use, confidence in and familiarity with the SC grew. However there is still a large group of students who experience technical difficulties when operating the SC. Will there be ways to shorten this period of adjustment? For a better diagnostic of students' knowledge and abilities, we developed a "competence model".

## 4. THREE DIMENSIONS OF A COMPETENCE MODEL

Since the PISA studies competence models are in vogue. Referring to the three-dimensional PISA-model with the "dimensions" Content (Numbers, Space and Shape, Change, …), Basic Competencies (Communication, Argumentation, Modelling, …) and Cognitive Activation (Reproduction, Connections, Reflections) we concentrated on the concept of functions and developed a model based on the three dimensions “Understanding of Functions”, “Tool Competences” and “Cognitive Activation”.

### 4.1 Understanding of Functions (UF)

Concerning the *Understanding of Functions,* we refer to a 4 level model. Each level is determined by a special knowledge students are expected to have.

Level 0: *Intuitive Understanding*.

Students

- know examples of functions;
- are able to represent real life situations as functions.

Level 1: *Conceptual Understanding*.

Students

- know different representations of functions (graphs, tables, equations);
- know properties of functions and recognize these properties in different representations.

Level 2: *Relational Understanding*.

Students

- know relations between functions, e. g. between the functions $f_n$ with $f_n(x) = x^n$, $n \in IN$.
- know relations between properties of functions, e. g. between the graphs of $f_a$ with $f_a(x) = \frac{1}{3}x^3 + a\cdot x - 1$, $a \in IR$.

Level 3: *Structural Understanding*.

Students

- are able to handle functions as objects, e. g. they see the function h with $h(x) = x^2 + \sin(x)$ as addition of the functions f and g;
- work with composition of functions, e. g. f o g or g o f

with $f(x) = x^2$ and $g(x) = \sin(x)$.

It seems that the SC does not play an important role on level 0. For this reason, we concentrate on levels 1 to 3 in the course of the competence model.

### 4.2 Tool Competence (TC)

Concerning the *Tool Competence,* we distinguish three different levels:

Level 1: Using the SC as a tool which produces static representations, e. g. graphing the function f with $f(x) = x^2 + 1$. We speak of a “Static Representation Mode” (StatRepMode);

Level 2: Creating dynamic representations, e. g. graphing the family of functions $f_a$ with $f_a(x)$ $a\cdot x^2 + c$. We speak of a “Dynamic Representation Mode” (DynaRepMode);

Level 3: Working with the SC on the symbolic level. We speak of a “Symbolic Mode” (SymbMode). This level integrates or is based on the levels 1 and 2.

### 4.3 Cognitive Activation (CA)

Concerning the *Cognitive Activation,* we use – like the PISA studies – three different levels

Level 1: Basic knowledge;

Level 2: Advanced knowledge;

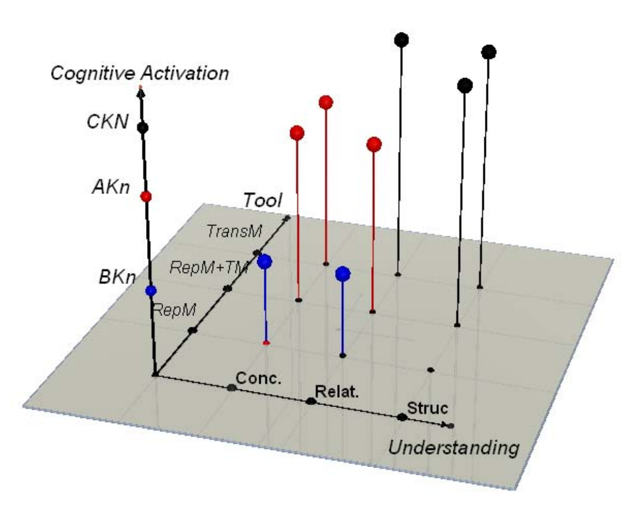
Level 3: Complex knowledge.

## 5. THE COMPETENCE MODEL

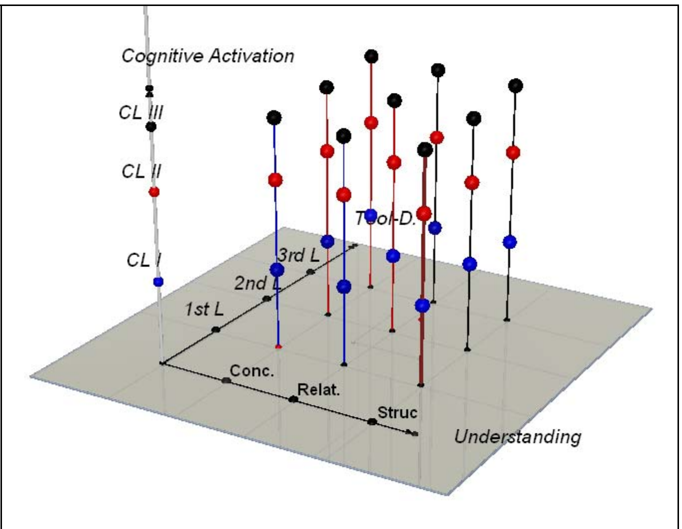
The table shows the relationship between the two “dimensions” UF (Understanding Functions) and TC (Tool Competencies). The relationship is established by special problems or tasks, showing on the one hand the level of understanding and on the other hand the way a student used the SC. This classification is realized by experts in a normative way, but is based on results of empirical investigations of the $M^3$-project and other empirical investigations in this area.

| | | | |
|---|---|---|---|
| SymbMode | Simplification of $f(x) = \frac{x^2 + x - 6}{x - 2}$ | Relation between f‘(x) = 0 an the extreme values of the graph of f | Symplification of $f(x) = \sin\frac{x}{2}\cos\frac{x}{2}$ |
| DynaRepMode | Graphing $f_a(x) = a \cdot x^2 + c$ | Relations between the graphs of $f(x) = x^n$, $n \in$ IN | Seeing h(x) = f(x) + g(x) as an addition of f(x) = sin(x) and $g(x) = x^2$ |
| StatRepMode | $f(x) = x^2 + 1$ | Seeing the relation between f(x) = … and f‘(x) = … | |
| | Conceptual Understanding | Relational Understanding | Structural Understanding |

The hypothesis is, that each of these 8 cells refer to a special level of CA (Cognitive Activation). It is still a hypothesis which has to be proved by empirical investigations. It also might be possible that there are two or three levels of CA referring to some or even all cells. This leads to a three dimensional competence model with the “points” or “cells” (UF, TC, CA). Pic. 1 and pic. 2 show the competence model with different numbers of “filled cells”. At the moment it is not clear how many “filled cells” are really needed.




Pic. 1. The Competence Model with 8 filled cells




Pic. 2. The Competence Model with 27 filled cells

Again: This model is still a theoretical model and has to be proved in the near future.

The question “CAS we can! – But should we?” is not answered with “yes” or “no”. It is rather transferred to the question of the validity of the competence model and this has to be evaluated by empirical investigations.

Especially the following questions have to be answered.

1. Is it possible to find or construct problems for each cell of the model for the classification of the knowledge and the abilities of a student?
2. How many cells have to be filled or how many triples (UF, TC, x), $x \in \{BKn, AKn, CKn\}$ exist for each pair (UF, TC)?
3. Can this model be empirically verified?
4. Is the model helpful for diagnostic reasons in the classroom?

The answers to these questions will – hopefully – be given in the course of the $M^3$-project in the near future.

**REFERENCES:**


Guin, D., Ruthven, K. and Trouche L. (Eds.) (2005), The Didactical Challenge of Symbolic Calculators, New York: Springer.

Weigand, H.-G. (2008). Teaching with a Symbolic Calculator in 10th Grade - Evaluation of a One Year Project, International Journal for Technology in Mathematics Education, Volume 15, No 1, 19-32