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Pilot Project "Integration of Media in Mathematics Tuition - M³" During The 2006/2007 School Year – Year 11

1. Background

1.1 The pupils

During the 2006/07 school year the pilot project – referred to in this report as the M3 project – was implemented in Year 11. A total of 732 pupils at 10 Bavarian grammar schools took part in this project. 412 pupils in 16 classes acted as the "pilot project classes", working with Voyage 200 and / or TI-Nspire. These units are referred to as pocket calculators (PC) in this report. In addition, 320 participants from 11 classes formed a control group for the purposes of quantitative statistical investigation. The pupils had different previous experiences; some pupils had been exposed to the PC in the previous Year 10, but other pupils came into contact with these systems for the first time during this project.

1.2 The teachers

The pilot project M^3 was mainly taught by teachers with little experience of tuition using computer algebra systems (CAS). The project teachers held two three-day meetings during which examples of possibilities and opportunities for PC use were discussed. The teachers jointly prepared a number of suggestions for a range of teaching units intended to highlight the possibilities of using PCs; during the year, the teachers were offered additional learning units by the coordinator (E. Bichler). However, there was no uniform overall concept according to which tuition was to be organised in all classes. The personal experience, attitudes and circumstances at the individual schools were too different for this to be possible.

1.3 Taught contents

In Year 11, calculus is taught (in Germany). The content taught was subdivided into the following:

- basic properties of functions (symmetry, monotonicity, variations in function terms and their impact on graphs, …)
- limits, continuity
- differentiability, derivation rules, derivation function(s)
- applications of differential calculus ("classical" functions discussion, extreme value problems)

1.4 The pilot project M^3 in the previous school year (2005/06)

During the 2005/06 school year, the pilot project was carried out in Year 10. The results of this project can be found in Weigand (2006, 2008). A number of pupils who took part in the pilot project in Year 11 had already been in a pilot project school in Year 10. However, the majority of pupils were first introduced to the calculator in Year 11.

2. Investigation questions:

The pilot project is based on the following questions:

1. Can any differences be ascertained in terms of core mathematical abilities (substitutions, interpretation of graphs, solving equations, working with tables, working with formulae) between the pilot and the control groups after one year?
2. Can different effects of PC use be ascertained with “good”, “average” and “weak” pupils?[1]
3. To what extent have pupils mastered the PC at the end of the year?
4. In what time frame do the pupils in the pilot project classes use the PC for problem solving?
5. With which representation levels (graphic, symbolic, numerical) do the pupils in the pilot project classes work?
6. What attitudes do the pupils in the pilot project classes develop in relation to the new tool?
7. What attitudes do the teachers involved in the project develop in relation to the new tool and teaching with the new tool?
8. Are there differences between boys and girls?

3. Test instruments

For the purpose of answering the 1st and 2nd questions:

A (classical) initial and final test – a test using paper and pencil but no calculator – was taken in pilot project and control classes.

For the purpose of answering the 3rd, 4th and 5th questions:

The pilot project classes also took a test using a PC in February 2007 and June 2007 in which they were asked to record their working methods with the PC in a questionnaire which they completed immediately *after* the test.

For the purpose of answering the 6th and 7th questions:

The staff responsible for teaching the pilot project classes completed an online questionnaire each month and an evaluation questionnaire at the end of the year. The pupils completed an online evaluation questionnaire which was intended to provide information regarding their experiences and their attitude to the PC.

For the purpose of answering the eighth question:

The initial and final tests were evaluated according to sex.

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[1] The performance criteria used relate to the results of the initial tests.

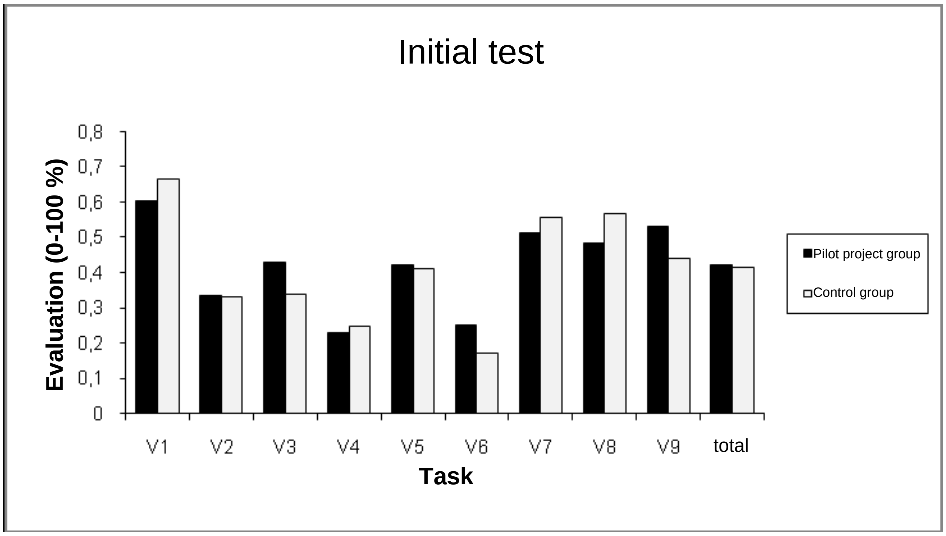
4. Evaluation of initial and final tests

4.1 The tasks

The tasks (see appendix) can be divided into the following groups:

1. Tasks 1 to 3: “classical” substitutions
2. Task 4: solving equations
3. Task 5: understanding concepts in root functions
4. Tasks 6 – 8: correlation between graph and term
5. Task 9: interpreting graphs

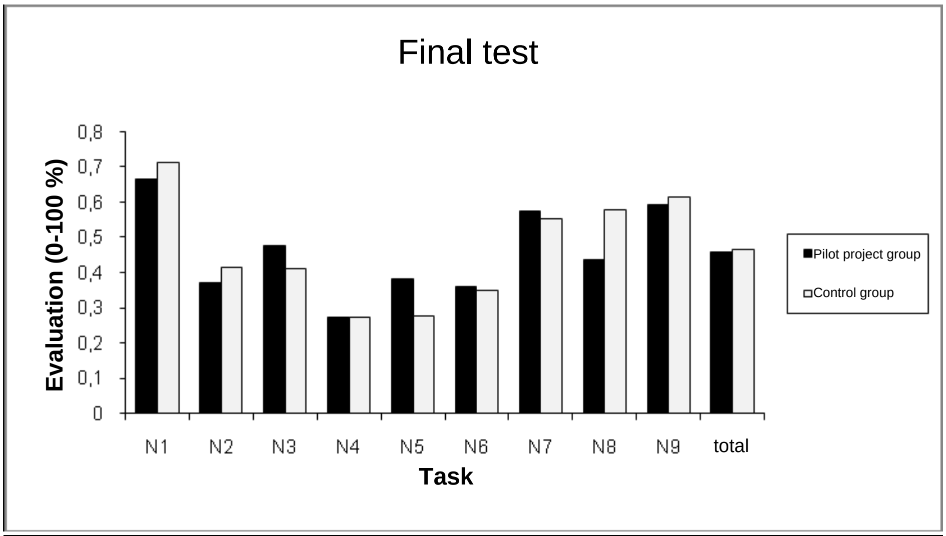
4.2 Results of initial or prior tests



The dark bars represent the results of the pilot project group, the light bars those of the control group. The percentage of correct answers is shown for individual tasks.

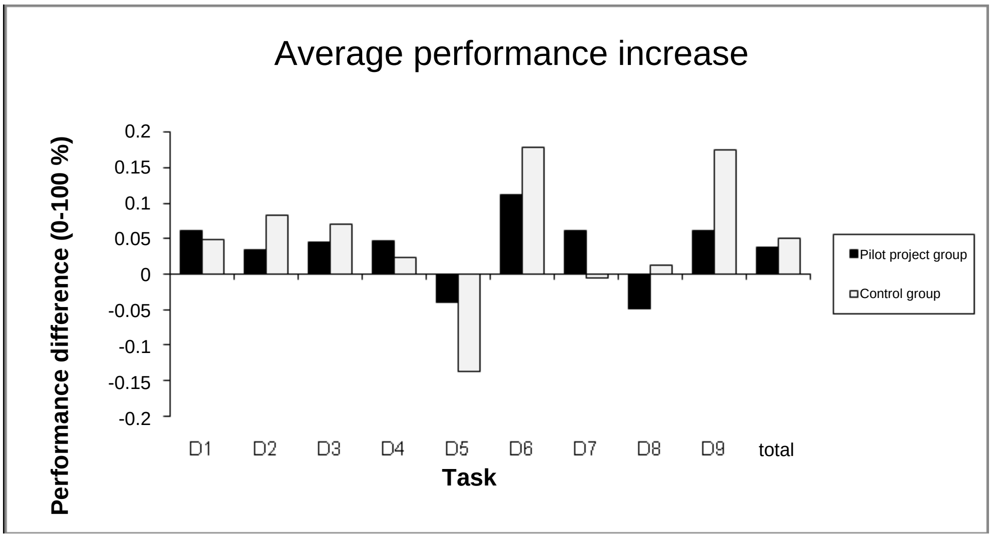
4.3 Results of final test

The final test was identical to the initial test. The results were as follows:



4.4 Comparison of results of initial and final tests

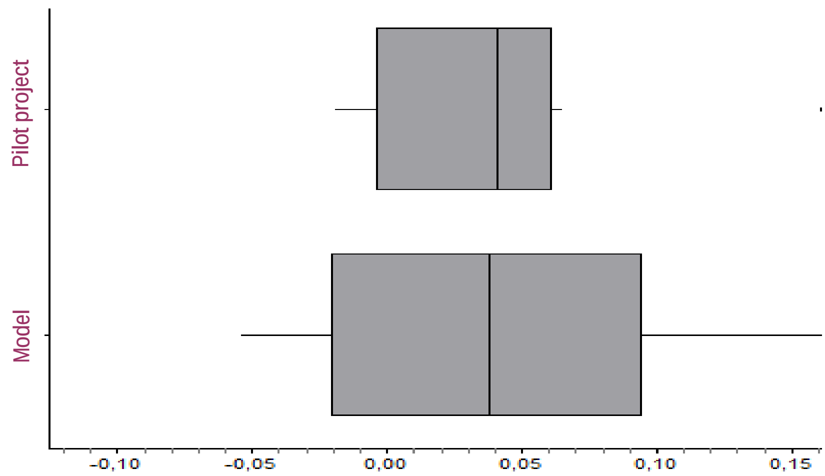
In the following diagram, the differences between the average scores achieved for each task in the initial and final tests for the M^3 and control groups are shown. The "average performance increase" is therefore measured for each task.



In tasks 5 and 7 the pilot project classes' results are significantly better than than those of the control groups (t-Test: 5: 0.01, 7: 0.02). However, in tasks 6 and 9 they are significantly worse (t-Test: 6: 0.01, 9: 0.01).

Overall there is a significant difference in the average performance increase between the pilot project and control classes. The average value of the average performance increase is 0.04 for the pilot project group and 0.05 for the control group. The deterioration of both groups in task 5 (understanding concepts of root function) is not surprising, as this aspect is not explicitly addressed in Year 11. The result – once again – indicates the necessity for permanent repetition of basic skills. For the comparatively worse result of the M^3 classes compared with the control classes (especially for tasks 6 and 9), there are two possible hypotheses. On the one hand it could be due to the fact that the pupils in the pilot project classes were no longer adequately motivated to tackle this type of "traditional" task with enthusiasm, as they had tackled much more interesting tasks during lessons – due to the PC. On the other hand the poor results of the pilot project classes when determining functional equations from specified graphs (task 6) could be due to the fact that the pupils in the pilot project class had seen a large number of graphs – compared with the control group – during the course of the year and were therefore overtaxed by the diversity. However, the pupils in the control class have probably worked more often with the sine function graph which had been introduced in Year 10.

If, however, the range of performance increases is considered, an interesting picture emerges.



Average value and range of performance increases in pilot project and control groups

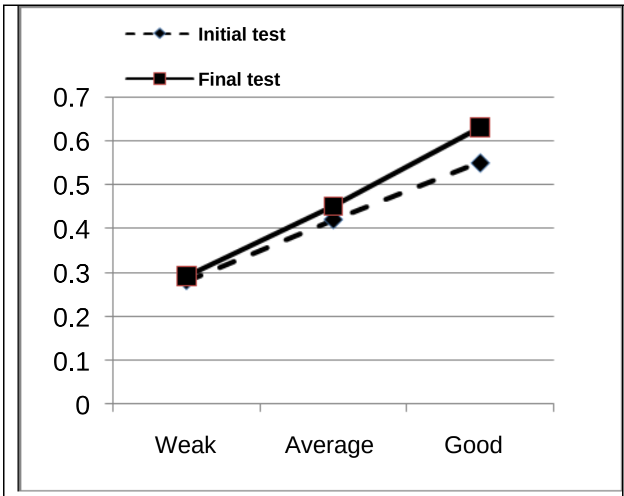
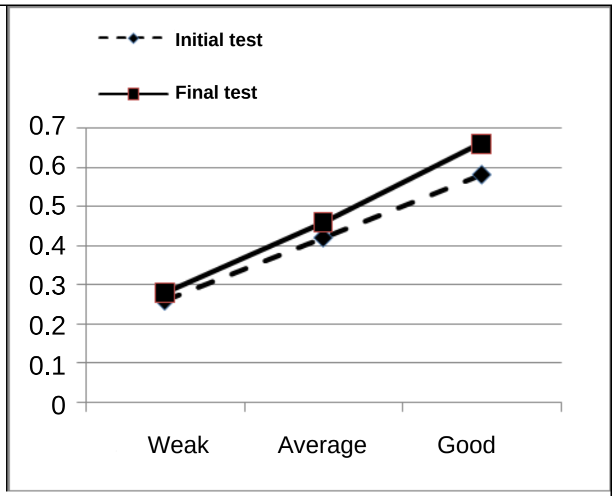
With an almost identical average value, it becomes apparent that the differences in performance are more varied with the pupils in the pilot classes than with the pupils in the control groups. Therefore, there are pupils in the pilot project classes who benefit more from PC use than pupils in the control classes. However, there are also pupils whose results deteriorate compared with the initial test.

The initial and final test investigation can be interpreted in a positive way for the pilot project classes, as there are no differences in terms of classical technical and manual abilities and skills (substitutions). However, this investigation has deflated hopes that the ability to interpret graphs and transfer between different forms of representation are automatically improved by the use of the PC. It remains questionable whether the pupils in the pilot project classes were being adequately challenged or motivated due to the largely classical nature of the test tasks.

4.5. Scores for "good", " average" and "weak" test participants

In accordance with the results of the initial test, we divided the test participants into "weak", "average" and "good".[2] The following result is produced when the performances of these groups are compared in terms of initial and final tests.

[2] The "good" pupils form the upper performance quartile, the "weak" pupils the lower performance quartile, and the "average" pupils are represented by the two central performance quartiles.

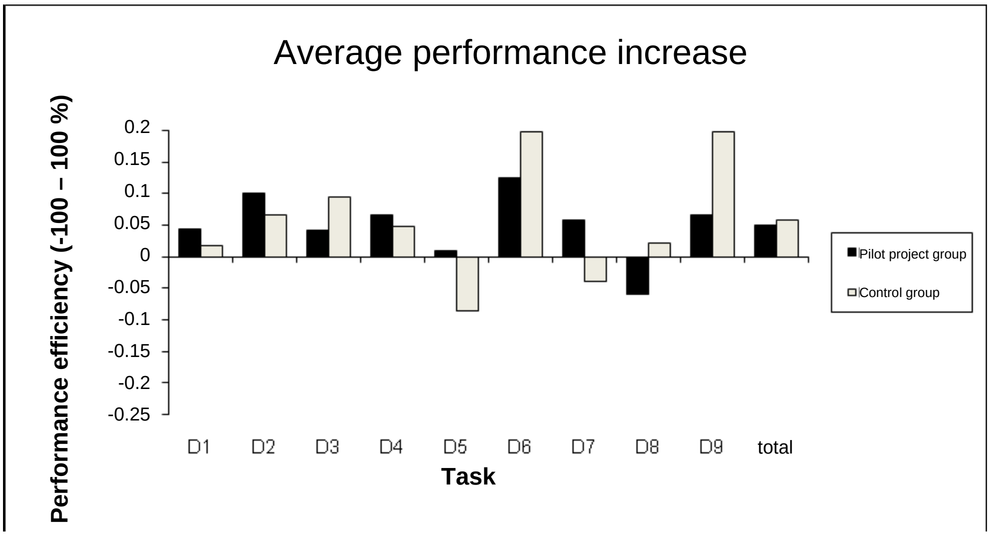
	
Pilot project group	Control group

Compared with tests carried out in recent years in Year 10 (cf Weigand 2006), different behaviour was demonstrated here. Whilst the “weak” pupils achieved a greater performance increase than the “average” and “good” pupils in Year 10, the “good” pupils – both in the control and pilot project groups – improved more markedly (by 8 percentage points) than the “average” and “weak” pupils (by 3 percentage points and 1-2 percentage points respectively) in the Year 11 test.

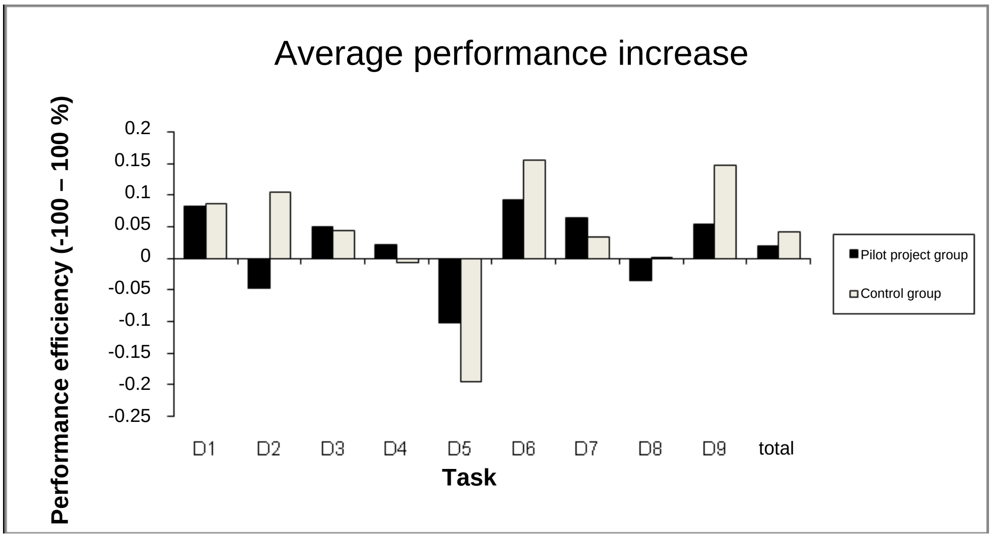
The differences between the “weak” and “good” groups can be found in the understanding of concepts (task 5) and the transfer between different forms of representation (between graph and equation - tasks 8 and 9)). The lack of performance increase in the case of weak pupils is attributable to the greater cognitive challenges posed by calculus, which may have taken some pupils to the limits of their capacities so that they were no longer able to follow lessons (“dropout effect”).

4.6 Comparison between girls and boys

We will now compare the increases in performance for individual tasks between boys and girls.



Performance differences for boys



Performance differences for girls

In the case of task 2 and task 5 (both tasks concern substitutions), the boys in the pilot project classes improve significantly more than the girls. Throughout the entire test, there are no (significant) differences between boys and girls.

5. The pocket calculator (PC) test

5.1 General introduction

In February and in June the pilot project classes took a test where they were allowed to use the PC. Use of the PC was optional for the pupils, i.e. they decided themselves whether or not they would use the calculator. The two tests consisted of four tasks each. In order to establish how calculators were used, we applied a new investigation method: the pupils completed a questionnaire on PC use immediately *after the test*, giving details of whether and how they used the calculator. This test was intended to answer the following questions:

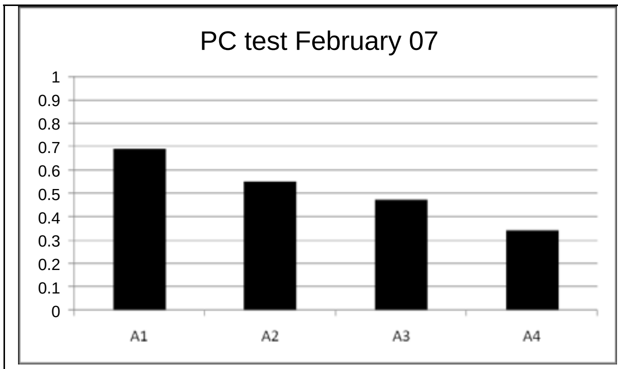
1. How do pupils use the calculator?

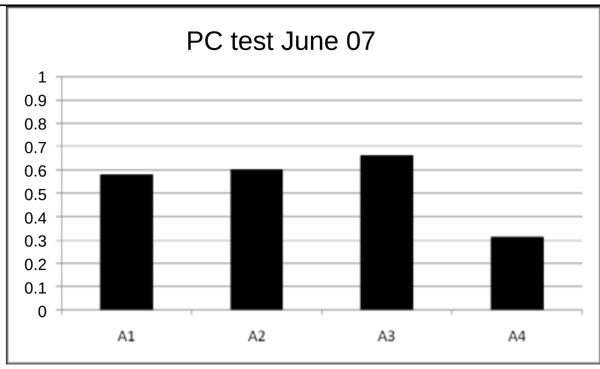
2. When – in terms of the phases in the problem solving process – do the pupils use the calculator?
3. Which functionalities (symbolic – graphic – numerical) do the pupils use?

In addition, the staff responsible for teaching the classes were presented with a questionnaire regarding the tasks immediately *before the test*, in which they were intended to provide details of the difficulties expected in terms of the tasks.

5.2 Results

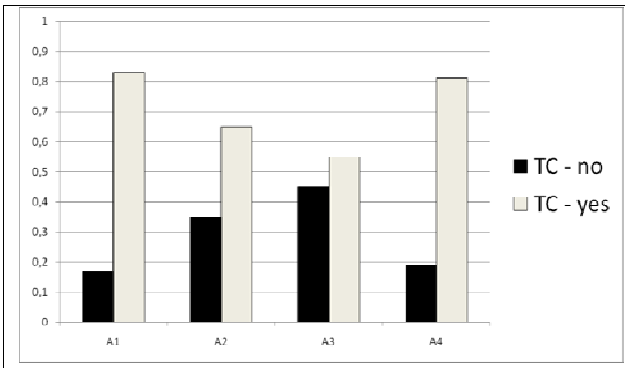
The test exercises are contained in the appendix. Use of the calculator during the test was optional for the pupils. The tasks could also be solved without using a calculator. Here are the percentages of correct solutions for individual tasks in February and June.



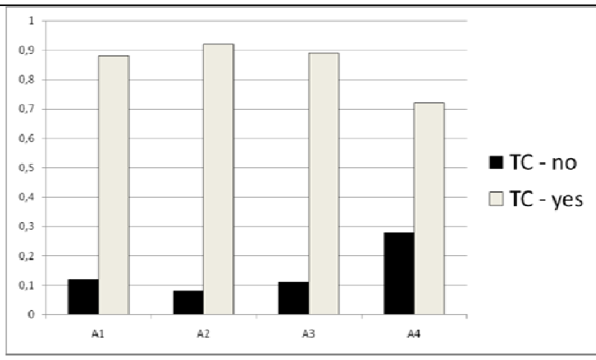


5.3 Actual use of the PC

The following diagrams show how many pupils used the PC during the tests in February and in June – according to their own statements:

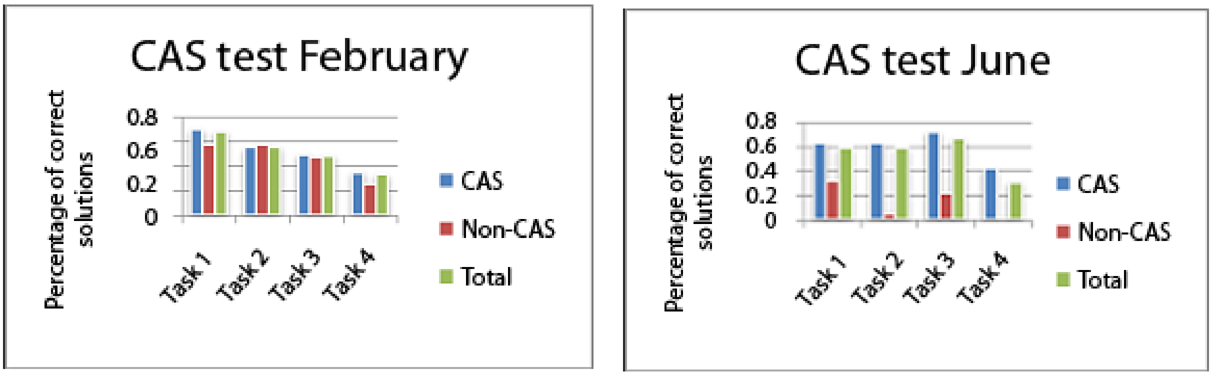


February test



June test

The difference between PC use in February and in June shows an increase in use of the calculator.
If the percentage of points achieved by both groups in the tasks is plotted, then the following picture emerges:

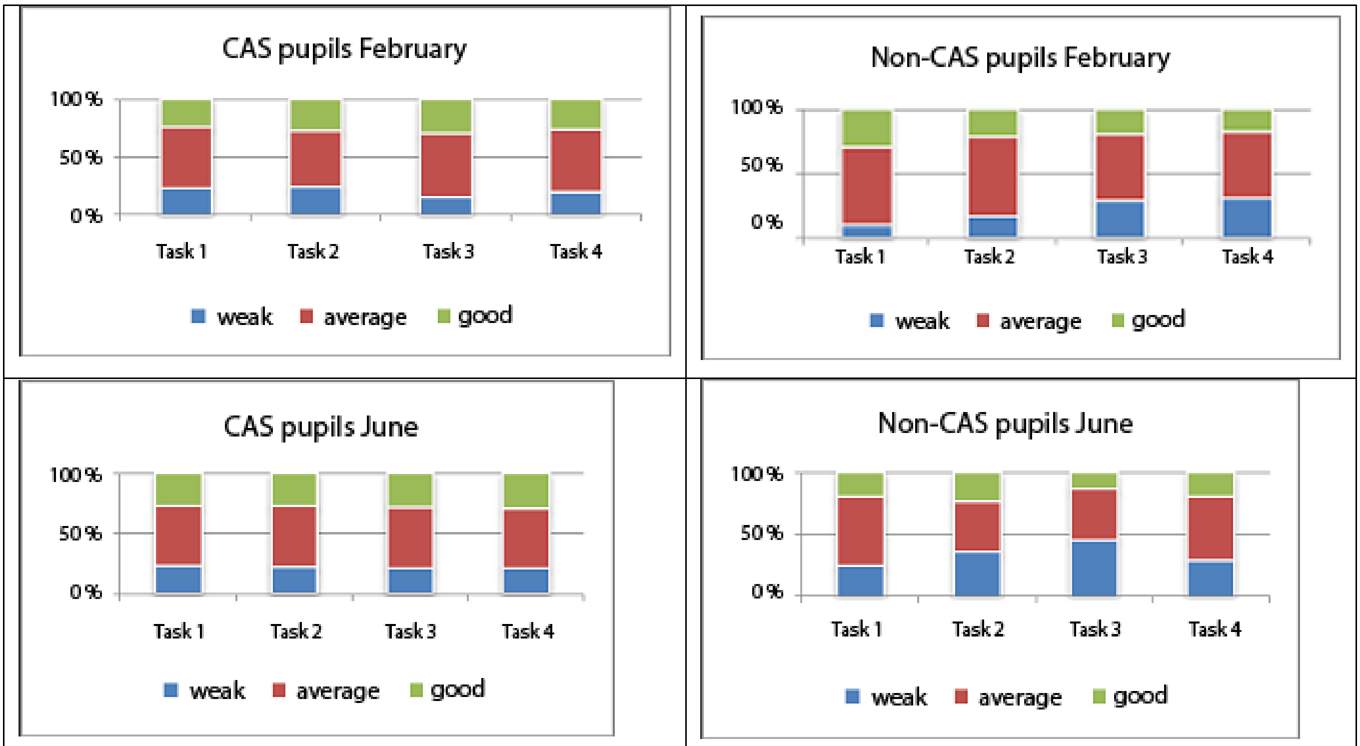


Average number of points achieved for tasks separated according to pupils who used CAS and those who did not use CAS (the overall average number of points is also plotted in each case)

This shows a clear development process between the February and the June test. Those pupils who used the PC in June when solving the tasks scored significantly better marks than those who did not use it. We attribute this to the fact that it takes a full school year for pupils to acquire adequate confidence in the PC, as well as knowledge of the benefits of its use as a tool when solving tasks, to be able to use these for the purpose of solving problems.

5.4 Results and ability groups

To which ability groups ("weak" – "average" – "good") are the pupils who used the PC or did not use the PC to be allocated? The following can be seen:

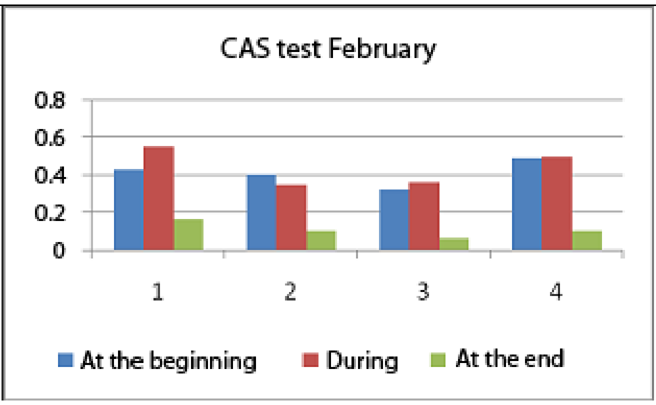


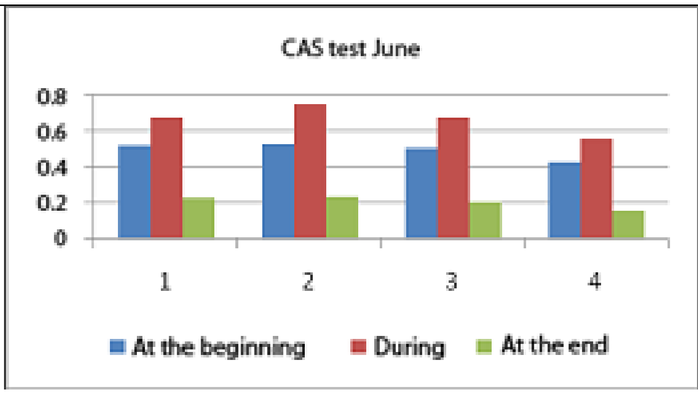
Proportion of pupils who used / did not use CAS in the June test according to the ability groups in the intial test.

It is apparent that the proportion of "weak" pupils in those who did not use the PC is slightly higher in some instances.

5.5 Point of use of the PC during the course of the problem solving process

The pupils also provided information in the questionnaire as to whether they used the PC at the outset of the solving process, and / or during solving and / or at the end.





Use of the PC during the course of the solving process (according to statements made by the pupils themselves)

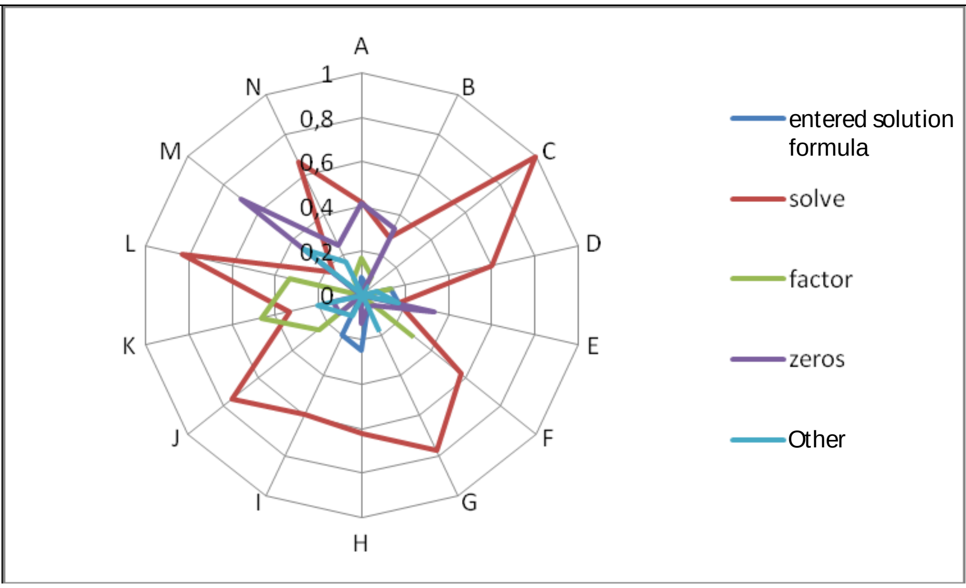
When pupils integrate the PC into their solving process, it is predominantly used at the beginning and during the solving process. If we compare the middle of the school year with the end, we can observe a clear increase in the frequency of positive responses to “during”. This allows us to conclude that the PC is more strongly integrated into the solving process by the pupils at the end of the school year. A slight increase can also be observed “at the end”, which makes us aware that checking the solution is gaining in importance.

5.6 Type of PC use

It appears that the pupils mainly use the symbolic and graphic possibilities of the PC. In contrast, pupils only avail themselves of numerical use to a very slight extent. However, it is noticeable that the type of use depends very strongly on the classes and indicates the significance of the teacher and his or her didactic approach. This can be explained by an example:

Task: Determine the maximum definition quantity of the function $f: x \mapsto \dfrac{2x^2 - 6x + 4}{x^2 + 2x - 2}$

The following diagram shows the use of appropriate calculator commands for this task for classes A to N.



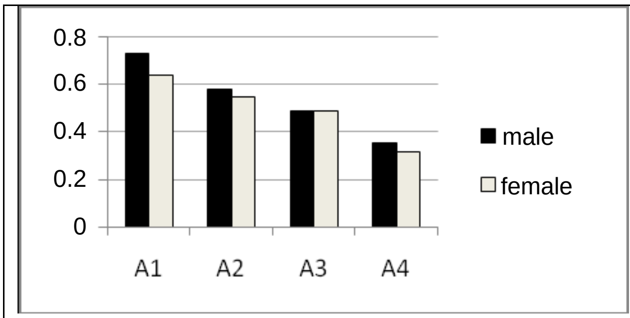
Strategies used during processing, itemised according to classes

There is a high degree of fluctuation between individual classes. In some classes all pupils used the command “solve”, in other classes the proportion is only 20 %. In one class a high proportion of pupils use the “zeros” command, which is otherwise not used very widely.

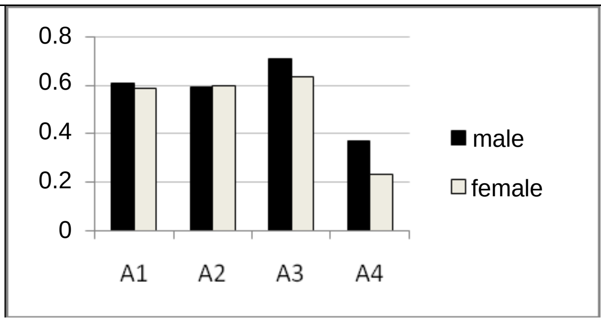
The pupils who did not use the TC usually mentioned several reasons. This can be interpreted as indicating that a lack of confidence in handling and using the PC was probably the main reason for a decision against use of a calculator.

5.7 Comparison of male and female pupils

When a comparison of male and female pupils is made, the following result emerges:



February test



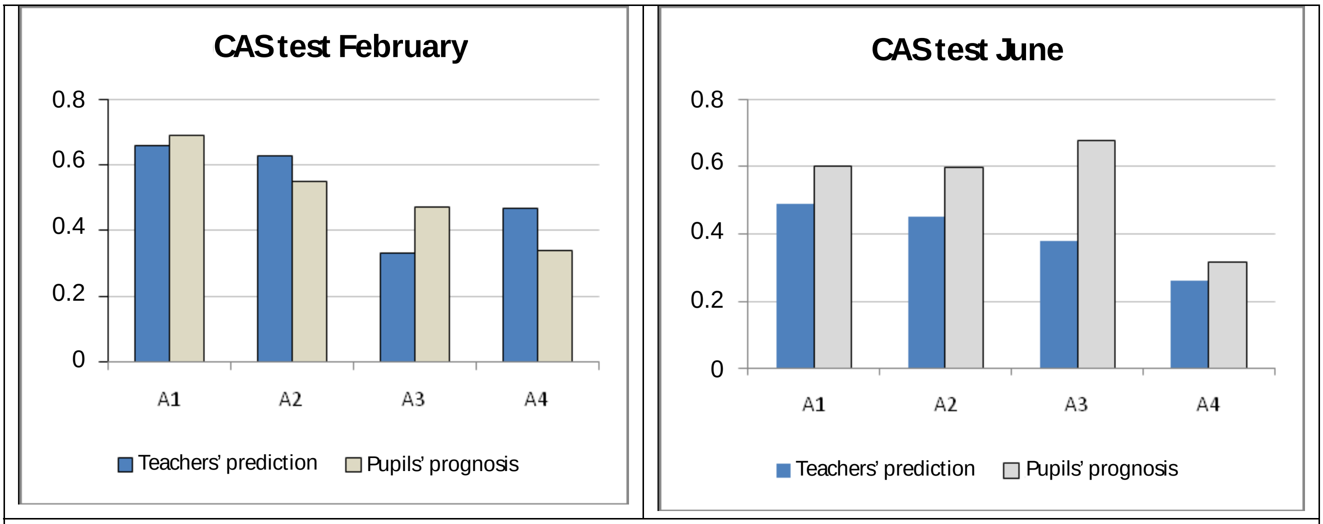
June test

The difference between boys and girls in the fourth task of the June test is somewhat surprising (although we do have an explanation)

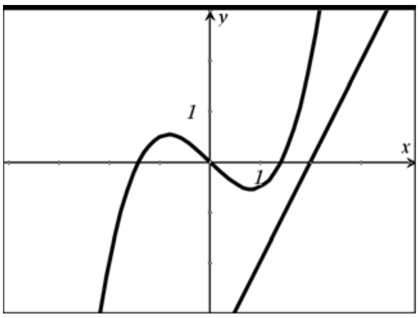
in terms of their willingness to tackle a difficult task at the end of the test.

5.8 Teachers' predictions

Before each test was carried out, the teachers provided an assessment of the extent to which pupils would solve the tasks. The results are as follows:



Comparison of teachers' predicted and pupil results in the PC tests



It is noticeable that the teachers rather overestimated the pupils in February but underestimated them right across the board in June. This is obviously particularly pronounced for task 3 of the June test.
This task involved proving whether a given straight line ($y = 2x-4$) is a tangent to the graphs of the function with $f(x) = 1/2x^3 – x$. (see adjacent figure). For the purpose of this task, the graph of both functions and the possibility of determining the intersection symbolically using the calculator appear to represent an important advantage of the PC.

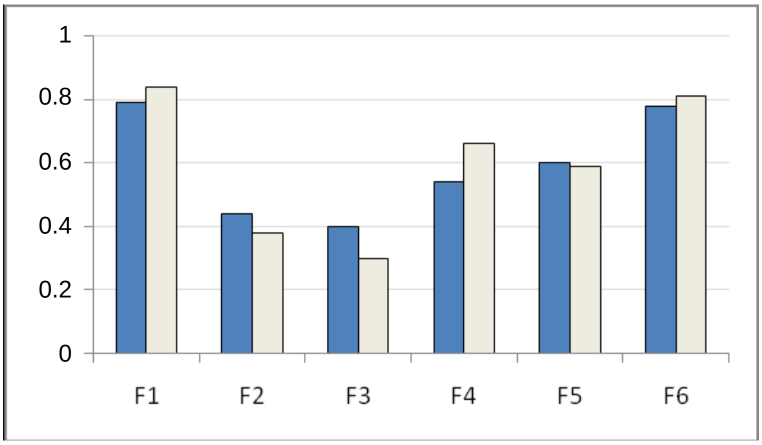
Task 4 of the February test is interesting for several reasons. Firstly, the proportion of those who looked at the graph at the beginning was very high (75 %). The intention was that the symmetry of the graph should be recognised. However, the task was only solved by 30 % of the pupils. The pupils had no strategy for solving this task. It can therefore frequently be observed that pupils resort to a standard tool box. A discussion of curves is initiated, derivatives and threshold values are calculated (against infinity) without seeing any bearing on the task. The difficulty of task 4 is probably above all due to the fact that the pupils have no algorithm to answer this question.
What is also interesting is the result of task 4 of the June test. This task involves a function type (logarithm function), which is not covered adequately at the Year 11 stage. The pupils are also not familiar with the derivation of the logarithm function (which is used for determining monotonicity). In this instance, it is therefore necessary to deploy the PC in “unknown territory” if one wants to reach a solution. This led to a solution of the task for almost a third of the pupils. What is surprising here is the significantly better scores achieved by male pupils.

5.9 Evaluation questions regarding the PC tests

The pupil questionnaires handed out at the beginning of both PC tests included the following general questions:

Q1	Did you find the PC was helpful when completing the tasks?
Q2	Did you experience any difficulties when recording the use of the PC in your solution in written form?
Q3	Did you have any difficulties operating the PC?
Q4	Would you agree with the statement that the PC gave you a feeling of security when completing the tasks?
Q5	When you think of the tuition you have received using the PC to date, did you find it interesting?
Q6	Have you ever used the PC to solve similar tasks in the past?

Here is a comparison of the results:



The dark bars represent the responses to the questionnaire of the February test, the light bars those for the June test.

Questions Q1 and Q4 show the trend that the PC became more familiar towards the end of the school year with the pupils finding it more helpful, giving them confidence when completing the tasks. Both the number of pupils who had difficulties when documenting the solutions (Q3) and those who reported technical difficulties (Q4) are comparatively high but decreased compared with the February test.

The responses of the pupils confirm that familiarity with the new tool requires a very long process of getting used to it. After one year of PC use, confidence in and familiarity with the PC grow. However there is still a large group of pupils who experience technical difficulties when operating the PC. Difficulties with the type and manner in which to document the solution decreased, but still remain at a high level. This latter point will continue to be a permanent challenge when working with the PC, as there is no algorithmic solution for the procedure.

6. Evaluation sheets for pupils and teachers

The teachers completed an evaluation sheet at the end of the school year and also completed a monthly online questionnaire. The pupils completed an online questionnaire.
The percentage listings are given in the appendix. The following core results can be derived from the interpreted data:

6.1 The teacher questionnaire

The evaluation questionnaire *at the end of the school year* yielded the following results – summarised in short form:

- The calculator was permitted in all classes – in the case of some assignments possibly only for part of the assignment.
- 60 % of the teachers are of the opinion that content has not changed compared with traditional tuition; only 40 % see changes in their teaching.
- 70 % of the teachers are of the opinion that the methodology of teaching has changed.
- Virtually all the teachers are of the opinion that pupils' chances of understanding the content have improved.
- Half of the teachers are of the opinion that it is essential that the PC is always available.
- The computer room is only used regularly by two teachers, and virtually all teachers prefer to work with the PC than in the computer room.
- With the exception of one teacher, all teachers expressed the opinion that they would like to continue to work with the PC.

The *monthly survey* produced the following results:

- The teachers assessed the pupils' “feelings” about the PC as being fairly positive
- 13 % of teachers use the PC in every lesson; 46 % use it in every other lesson.
- The PC is used above all for function plotting (88 %), for solving algebraic equations (73 %) and for substitutions (65 %). Graphic equation solving takes place relatively rarely (35 %).
- The PC is mainly used for practicing (75 %) and for visualisation (75 %).
- When the PC is used, work is approached independently (71 %), alone (56 %), or in groups (50 %).
- Considerable importance is attached to the calculator as a checking instrument in the hand of pupils.
- Teachers regard the main cause of the problems experienced with the PC as being a lack of the knowledge necessary for operating the unit.

The PC is regarded by teachers as being a beneficial new tool. It changes teaching methodology, provides an opportunity to integrate new content or at least for new treatment of old content and serves pupils as a visualisation and checking instrument. The calculator is generally used in a carefully considered and very restrained way. As well as using the PC for function plotting, it is also used for equation solving and substitution. The computer algebra component of the PC is regarded as an important element. Numerical and graphic methods appear to play a more subordinate role.

6.2 The pupil questionnaire

An evaluation questionnaire *handed out at the end of the school year* produced the following picture of the pupils in *the pilot project classes* (figures are quoted as percentages; to facilitate orientation, responses which tend overall towards positive or negative are highlighted in bold face):

		definitely correct	correct	no difference	not correct	definitely not correct
1	Lessons using pocket calculators were more interesting than lessons used to be.	**9**	**44**	30	10	7
2	Lessons became easier as the result of using pocket calculators	**13**	**43**	24	14	6
3	Lessons using pocket calculators were more varied	**13**	**58**	13	11	5
4	In lessons using pocket calculators, I learned more than in previous lessons.	6	18	31	**26**	**19**
5	I found lessons using pocket calculators more enjoyable.	**6**	**35**	30	15	14
6	Lessons using pocket calculators introduced me to a completely new side of mathematics.	5	22	29	**24**	**20**
7	I used the pocket calculator outside lessons on using pocket calculators.	**13**	**30**	11	**23**	**23**
8	I would like to continue to work with pocket calculators.	**20**	**35**	14	15	16
9	I often used a pocket calculator to do my homework.	**17**	**41**	10	18	14
10	I found pocket calculators helpful.	**24**	**49**	13	9	5
11	I would recommend pupils to attend a class on pocket calculators.	**12**	**34**	19	23	12
12	Pocket calculators give me a	**23**	**44**	13	14	6

	feeling of confidence when solving tasks.					
13	I frequently did not know how to operate the pocket calculator.	**17**	**24**	11	**33**	**15**
14	I feel less well prepared for the next school year than the pupils in the parallel classes.	**13**	**22**	21	**21**	**23**
15	I was often not clear what documentation was expected when solving a task with the pocket calculator.	**10**	**30**	21	**25**	**14**
16	It's OK to use a pocket calculator during lessons, but not for homework.	16	14	17	**24**	**29**
17	I did not find that using pocket calculators for homework was a problem.	**23**	**33**	20	14	10

The following results are worth mentioning:

- The lessons were regarded by the majority of pupils as being varied (questions 1, 2, 3 and 5). What is new is that they are regarded as being easier.
- A large majority see pocket calculators as being helpful and as providing security when solving tasks. This is confirmed by the conclusion, drawn on the basis of the PC tests, that the PC had become an integral tool by the end of the school year.
- Approximately a quarter of pupils experience problems with written exams (questions 16 and 17). A tuition plan using CAS must address this point to reduce the very high proportion of pupils experiencing problems of this nature.
- Questions 8 and 11 are interesting compared with previous years. A far greater proportion of pupils would recommend that their fellow pupils to attend a PC class than was previously the case. We attribute this to the fact that in this phase of the pilot project the pupils faced the prospect of being able to continue to use the PC after Year 11, whereas previously use was limited to one school year only.

- A polarisation into two approximately equal proportions of attitudes reflecting approval and rejection is indicated in respect of the following aspects:
    - Operating problems (question 13) and associated preoccupation with the PC outside lessons (question 7)
      An additional question is the extent to which this is the result of tuition and the extent to which this is related to pupils' performance.
    - Documentation of solutions (question 15)
      That two fifths of pupils do not know what documentation is expected when using the PC remains a surprisingly high proportion for the end of the year. This aspect was obviously not adequately dealt with in lessons (at least in some instances).
    - Preparation for the next school year (question 14)
      This question is certainly difficult to judge from the pupils' point of view. However, the dichotomy indicates that some pupils are still apprehensive. This is impacted on by the fact that not all pupils can use the PC again in the next school year.

7. Use of pocket calculators for class tests

The tasks set for class tests at least give some insight into the tasks which were also dealt with during lessons. There follows a list of a range of problems which – compared with traditional tasks – contain a new component in that pupils are not able to solve them without using a calculator, or arose as a result of using a calculator.

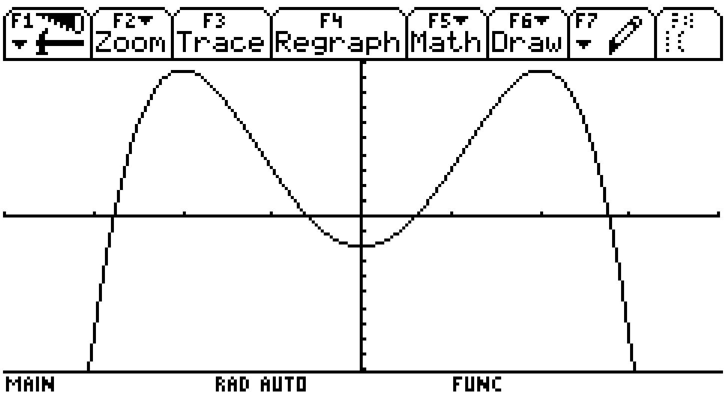
New tasks:

1. Polynomial function

Itemise all roots of f(x) including their multiplicities.

$$f(x) = -x^6 + 4x^4 - 8x^3 + 15x^2 - 16x + 6 \quad ; D_f = IR$$

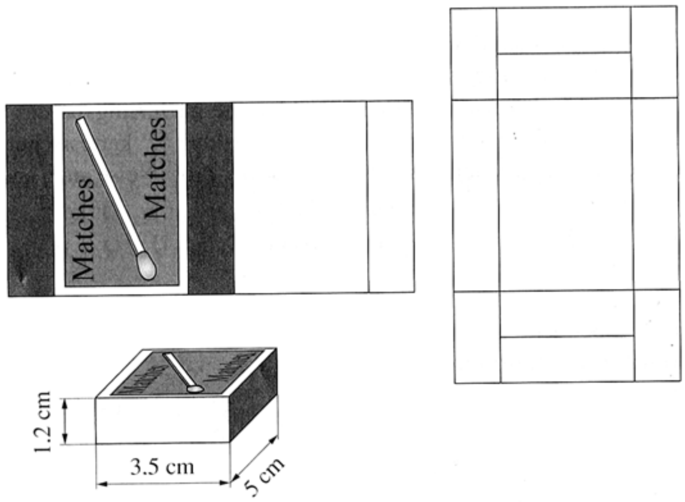
2. Ascertainment of a functional term



Ascertain the functional term of a polynomial function with approximately the graph indicated ($x \in [-4;4]$; $y \in [-10;10]$).
Justify your choice of approach, state the coordinates of the supporting points used and the resulting functional term and calculate the functional value for $x = -1$.
Interpret the special features indicated by the coefficients of the found polynomial function, in conjunction with the graph.

3. Extreme value task

Our match boxes have the dimensions $L = 5\,\text{cm}$, $B = 3{,}5\,\text{cm}$, $H = 1{,}2\,\text{cm}$. They are folded in accordance with the displayed construction instructions.
Using the same length, the same volume and the same type of construction, could we manage with less material (i.e. area)?



4. Applying related rates ("related rates problems")

A kite is in the air at an altitude of 400 feet. The kite is being blown horizontally at the rate of $10 \frac{feet}{\sec ond}$ away from the person holding the kite string at ground level.

At what rate is the string being paid out when 500 feet of string is already out?

(*Assume that the string forms a straight line.*)

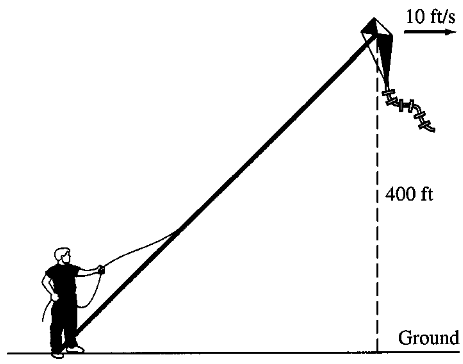
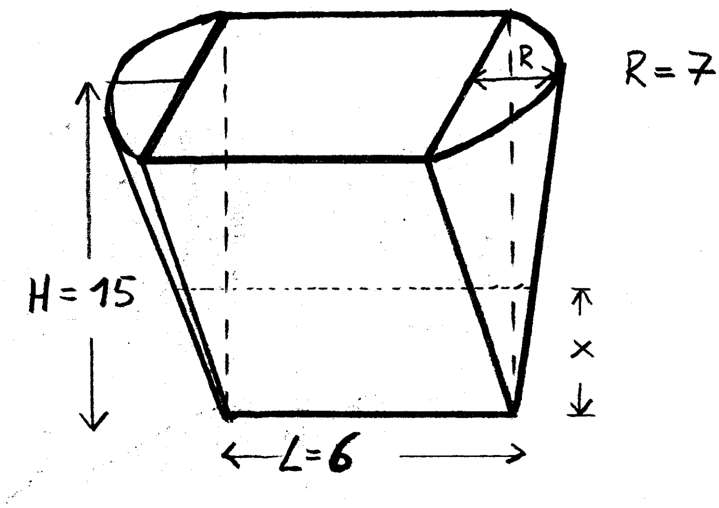
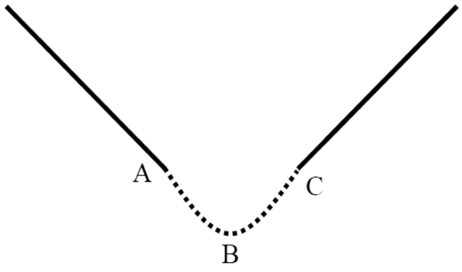


FIGURE 4.1.18 The kite of Problem 45.

A ("manual") coffee filter consists of a prism with two laterally displaced semi-cones. The dimensions of the filter can be taken from the drawing (dimensions in cm) below. 2 cm³ of coffee flow out of the filter per second.
What is the level speed of the coffee at the moment when the liquid level x in the filter is exactly 2 cm?



5. Extreme value task



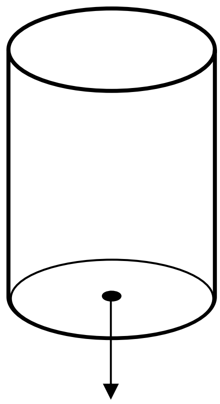
Two straight stretches of road are to be connected by a bend between points A and C.
The stretches of road form an angle of 45° to AC in each instance. Stretch [AC] is 60m long; point B is located at a distance of 20m from [AC].
At transitional points A and C, the bend should not be sharp; moreover, the curve should be minimal at these points.
Calculate an appropriate polynomial function describing the curve.
Describe the path to your solution in a clear and intelligible fashion.

Various interpretations:

1. Derivation as local linear approximation

a) When can a function *f* at point x_0 be called local linear approximative? State our definition (*not the graphic interpretation*).
b) In doing this, demonstrate that an *x*-$_0$-located differentiable function *f* at x_0 is also constant.

2. Derivation as modification rates



In a vessel shaped like a circular cylinder, there are 600 litres of water. A drainage hole is opened in the base; the vessel then empties in 60 minutes. The drainage hole is opened at point in time $t = 0$. The quantity of water still located in the vessel at t minutes is described by the function $V : t \mapsto \frac{1}{6}(60-t)^2$

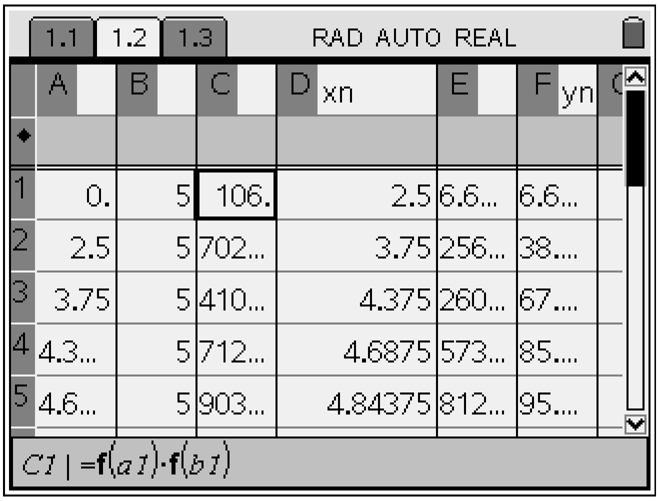
a) Determine the average rate at which the water quantity in the vessel changes between $t = 15$ and $= 45$.
b) Determine the local rate of change of the quantity of water at point in time $t = 15$.
c) Comment on the algebraic sign of the values calculated.

Numerical methods

Roots:

In lessons, we discussed an approximation procedure for root determination and implemented it by means of spreadsheet analysis when applying roots.

The function $f: x \mapsto x^3 - 4x + 1$ is now observed with maximum definition quantity. The following screenshots show an extract from our table for the specified function and the start interval [0;5].
If you look at column D, it could be assumed that a sought root is located close to 5.
Is that correct? If not, where is the error?

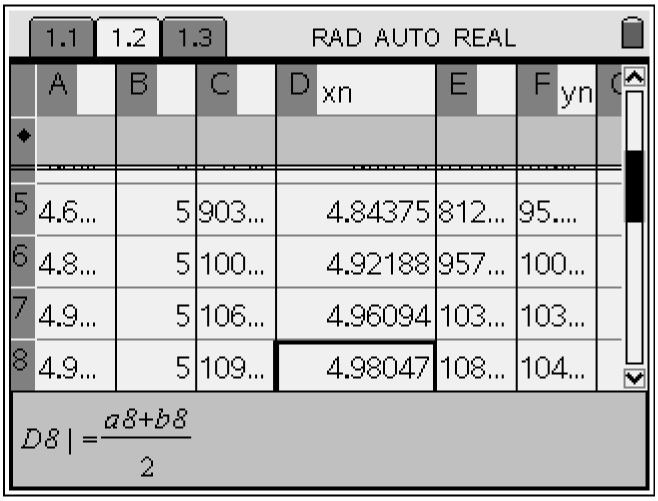


RAD AUTO REAL

	A	B	C	D xn	E	F yn
1	0.	5	106.	2.5	6.6...	6.6...
2	2.5	5	702...	3.75	256...	38....
3	3.75	5	410...	4.375	260...	67....
4	4.3...	5	712...	4.6875	573...	85....
5	4.6...	5	903...	4.84375	812...	95....

C1 | =f(a1)·f(b1)

Screen Shot



RAD AUTO REAL

	A	B	C	D xn	E	F yn
5	4.6...	5	903...	4.84375	812...	95....
6	4.8...	5	100...	4.92188	957...	100...
7	4.9...	5	106...	4.96094	103...	103...
8	4.9...	5	109...	4.98047	108...	104...

D8 | $=\frac{a8+b8}{2}$

Screen Shot

8. Summary of results:
We will now summarise the core results of this one-year school trial once again.

- **Initial and final test**. No differences result between pilot project and control classes for the initial and final test – taken in the traditional way with pencil and paper, without technological aids. In view of future PC use, this is pleasing on the one hand, as "traditional work" (substitutions, solving equations) using pencil and paper is not being forgotten. On the other hand, hopes have not been fulfilled that pupils in the pilot project classes would improve to a greater degree in terms of dealing with and interpreting graphs than pupils in the control classes. The question remains as to whether pupils in the pilot project classes might possibly not have been adequately challenged or motivated as the result of the largely traditional nature of the test tasks.

- **Polarisation**. When working with new technologies, polarisation occurs in that some pupils benefit greatly from PC use, whereas for other pupils, PC use inhibits performance or even decreases performance.
  This polarisation is also demonstrated by the pupils' assessment of their own work with the PC. Two thirds of pupils are of the opinion that the PC was helpful and made them more secure and they classify lessons as "interesting". Approximately one third of pupils do not share this view. This polarisation also comes to light in terms of the statements made regarding operating problems.

- **Type of calculator use**. When the calculator is used, it tends to be used at the outset and whilst solving the problem rather than for checking purposes. The unfamiliar presentation of problems leads to – sometimes aimlessly – testing familiar routines. The reasons for non-use of the calculator are on the one hand the uncertainty of pupils regarding technical handling of the unit and on the other hand a lack of knowledge regarding use of the unit in a way which is appropriate for the particular problem.

- **Period of adjustment**. It is surprising that it took almost a year to establish adequate familiarity with this tool for pupils to use it during exams. With familiarity, a more considered use of the tool is demonstrated. Towards the end of the school year, it is regularly used at the beginning of the task (graph as overview), as well as at the end to check the solution.
  Solution documentation Pupils were presented with problems in relation to how to record solutions when using CAS. This is confirmed by both the evaluation questionnaires at the end of the year and the PC tests. It is necessary to cover this topic to a greater extent during lessons.

- **Teachers' opinions**. The teachers see the PC as a helpful tool for lessons which has an impact on teaching methodology and social forms. Teachers mainly use it for functional plotting and as a symbolic tool (substitutions, solving equations). Graphic and numerical procedures for problem solving play a less important role.

- Pupils' opinions. Pupils see lessons with CAS as positive, varied and interesting. They invariably want to continue to work with the PC.

Appendix

Task 1:

Determine the maximum definition quantity of the function $f: x \mapsto \frac{2x^2 - 6x + 4}{x^2 + 2x - 2}$.

When did you use the PC during the course of solving the subtask?	At the beginning	During	At the end
	0.35	0.42	0.13

Multiple answers were possible for all questions.

When you used the PC, what did you do with it?	Entered solution formula	solve	factor	Zeros	Other
	0.06	0.52	0.10	0.16	0.07

If you did not use the PC, what were the reasons?	Would not have been any help	I was quicker manually	Did not know where	It did not occur to me	Other
	0.10	0.12	0.13	0.00	0.06

Task 2:

Determine the limit value $\lim_{|x| \to \infty} \frac{x^2 - 7x + 1}{-3x^2 + 5x}$.

When did you use the PC during the course of solving the subtask?	At the beginning	During	At the end
	0.34	0.28	0.10

If you used the PC, what did you do with it?	Looked at graph	Limit	Looked at value table	Calculated function values	Other
	0.17	0.47	0.01	0.03	0.05

If you did not use the PC, what were the reasons?	Would not have been helpful	I was quicker manually	Did not know where	On the basis of the terms one can recognise the limit immediately	Preferred to calculate manually	Other
	0.07	0.08	0.10	0.04	0.10	0.01

Task 3

The family of functions $f_a : x \mapsto \frac{a \cdot x + 2}{x^2 - 4}$ is given with $a \in R^+$. Investigate the characteristics of f_a at point x =2

When did you use the PC during the course of solving the subtask?	At the beginning	During	At the end
	0.24	0.31	0,08

When you used the PC, what did you do with it?	Looked at graph	Value table	Limit	h-method with PC	one-sided limits	Factor	Other
	0.28	0.02	0.13	0.02	0.03	0.07	0.04

If you did not use the PC, what were the reasons?	Would not have been helpful	I was quicker manually	Did not know where	Preferred to calculate manually	Did not occur to me	Other
	0.10	0.09	0.16	0.09	0.04	0.06

Task 4

Provide a justified assumption regarding the symmetry of the graph of the function $f : x \mapsto \frac{x^2 - 4x + 9}{x^2 - 4x + 5}$.

When did you use the PC during the course of solving the subtask?	At the beginning	During	At the end
	0.43	0.37	0.09

If you used the PC, what did you do with it?	Looked at graph	Looked at values table	Converted terms	Other
	0.75	0.06	0.06	0.05

If you did not use the calculator, what were the reasons?	Would not have been helpful	I was quicker manually	Did not know where	Preferred to calculate manually	Did not occur to me	Other
	0.02	0.03	0.07	0.03	0.01	0.04

June PC test

Task 1:

Determine the characteristics of the function $f : x \mapsto \frac{x-1}{x^2-1}$ by the definition interval x = 1.

When did you use the PC during the course of solving the subtask?	At the beginning	During	At the end
	0.50	0.64	0.20

When you used the PC, what did you do with it?	Entered solution formula	solve	Limit	Graph (s)	Other
	0.07	0.30	0.56	0.51	0,11

If you did not use the PC, what were the reasons?	Would not have been helpful	I was quicker manually	Did not know where	Did not occur to me	Other
	0.19	0.16	0.15	0.03	0.10

Task 2:

Determine the location and type of extreme value $f : x \mapsto \frac{x^2-2}{(x+2)^2}$

When did you use the PC during the course of solving the subtask?	At the beginning	During	At the end
	0.49	0.72	0.20

If you used the PC, what did you do with it?	Looked at graph	Derivation	solve	Calculated functional values	Other
	0.49	0.73	0.47	0.35	0.09

If you did not use the PC, what were the reasons?	Would not have been helpful	I was quicker manually	Did not know where	Preferred to calculate manually	Did not occur to me	Other
	0.10	0.07	0.09	0.09	0.03	0.06

Task 3

Is the straight line with the equation y = 2x-4 tangent to the graphs of $f: x \mapsto \frac{1}{2} \cdot x^3 - x$?

When did you use the PC during the course of solving the subtask?	At the beginning	During	At the end
	0.48	0.64	0.19

If you did use the PC, what did you do with it?	Looked at graph	Value table	Solve	Tangent	Derivation	Other
	0.61	0.03	0.37	0.10	0.43	0.05

If you did not use the PC, what were the reasons?	Would not have been helpful	I was quicker manually	Did not know where	Preferred to calculate manually	Did not occur tome	Other
	0,10	0,09	0,10	0,08	0,04	0,04

Task 4

State the monotonicity characteristics of the function $f: x \mapsto 1 - (\ln x)^2$ for x > 0.

When did you use the PC during the course of solving the subtask?	At the beginning	During	At the end
	0.40	0.55	0.15

If you used the PC, what did you do with it?	Looked at graph	Looked at values table	Derivatives	Other
	0.48	0.06	0.42	0.13

If you did not use the PC, what were the reasons?	Would not have been helpful	I was quicker manually	Did not know where	Preferred to calculate manually	Did not occur tome	Other
	0.13	0.06	0.16	0.05	0.06	0.10

Literature:

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Weigand, H.-G., Teaching with a Symbolic Calculator in 10th Grade - Evaluation of a One Year Project, International Journal for Technology in Mathematics Education, Volume 15 (2008), No 1, 19-32