

TEACHING WITH A SYMBOLIC CALCULATOR – THE FIRST YEARS OF A LONG-TERM PROJECT

Hans-Georg Weigand

University of Wuerzburg, Germany

A six-year project was started in autumn 2003 to test the use of scientific calculators in grammar schools in Bavaria (Germany). The aim of the project was to answer the following questions:

- *How have basic mathematical skills (e.g. algebraic transformations, solving equations, working with tables and formulas) changed?*
- *How have the questions posed in examinations changed when the students are allowed to use a scientific calculator (with CAS)?*
- *How have the students evaluated the use of the new tool?*
- *How have teaching styles and methods changed in mathematics lessons?*

The project should prove the practicability of using symbolic calculators in regular maths lessons. This article presents the results of the first three years of the project.

THEORETICAL BACKGROUND

Even though students in many countries are allowed to use symbolic calculators (with CAS) for schoolwork and in examinations, they have only been partially integrated into mathematics teaching worldwide (in contrast to graphics calculators, which have become mandatory in the majority of countries). There is a variety of reasons for this: they involve the tool itself (i.e. complexity of use, low resolution of the graphics screen, lack of a pedagogical tool), the views of the teachers (i.e. insufficient familiarity with the tool, concern about low abilities of the students, importance of traditional mathematics) and also the curricula (i.e. inadequate integration of the new tool into the goals of mathematics lessons). Moreover, the complex area of integrating the technology has been underestimated (see Trouche 2005).

In recent times the “theory of instrumental genesis” has been developed to describe the process of integrating calculators into mathematics education. The focus of this theory is to develop learning environments or an “instrumental orchestration”, in which the calculator (the artefact), or tool, changes into an *instrument* that’s useful for solving problems and that mediates between the user and the mathematical contents (see Drijvers & Herwaarden 2002, Artigue 2002, Drijvers & Gravemeijer 2005).

In the past years, many empirical investigations concerning the use of CAS or scientific calculators (with CAS) in the teaching of mathematics have been published (see Lagrange 1999, Peschek & Schneider 2002, Weigand & Weller 2001, Guin et.

al. 2005). These studies show that the use of a CAS brings a greater meaning to work with diagrams, reinforces experimental work in which the assumptions were obtained through systematic testing and that CAS appears to bring an increase in automatic work as well as computer cooperative forms of work. However, many investigations only looked into the use of the computer for a few weeks (Schneider 2000, Barzel & Möller 2001, Drijvers 2003, Pierce & Stacey 2004), and thus do not show the long-term effects on the knowledge and ability of the students.

In the following, the evaluation of the first three years of the project on the use of SC in the 10th school year of Bavarian grammar schools is presented.

THE TEACHING PROJECT

In the school year 2003/04, a total of 137 students from six Year 10 classes (referred to in the following as SC-classes) in three Bavarian grammar schools used the TI Voyage 200 (referred to here as a symbolic calculator or SC) in lessons and exams for one year. Four Year 10 classes (121 students in total) were included as control classes. The project was repeated in the school year 2004/05 with ten Year 10 classes (118 students used the SC and 126 students were in the control classes) and again in the school year 2005/06 with twenty-two Year 10 classes (257 SC and 145 in the control). The project will continue in subsequent years in Years 11, 12 and 13.

The following topics were taught: calculating with powers and power rules, power functions, the exponential and logarithmic functions, measurements of circles, trigonometry, the volume and surface area of cylinders, cones and spheres.

The study was intended to answer the following questions:

1. What changes in core mathematical abilities of the students in SC-classes could be identified after one year? (E.g. formation of terms, interpretation of graphs, solving equations, working with tables and formulas.)
2. Does – as is frequently claimed – the difference between ‘good’ and ‘poor’ students increase with the use of the SC?
3. What kinds of attitudes or beliefs develop among students using the SC? Are there differences between male and female students?
4. How do examination questions change as a result of the use of the SC?
5. How are the capabilities of the students developed in learning to use the SC effectively?
6. How do the teachers evaluate their teaching lessons and their change through the use of the SC themselves?

TEST APPARATUS

Questions 1 and 2 were answered by a pre- and post-test design.¹ All tests were taken using paper and pencil, the use of SC was not allowed. The questions were about basic mathematical skills like the manipulation of terms but there were also questions involving interpretation of graphs and transfers between different representations. To answer the third question, a questionnaire was developed with answers given according to a five-point scale and questions allowing open answers. For the fourth question, external experts evaluated the regularly written exam questions of the SC-classes with respect to the expected competencies of the students and possible solving strategies (while using a SC).

Research question four should give hints as to whether the teachers integrated more questions – compared to “traditional” examinations – concerning process-oriented competencies like problem solving or modelling.² To answer the fifth question, the SC-classes had to pass a post-test with SC as well.³ This test was a new feature in school year 2005/06 and was not given the years before. For the sixth question, the teachers of the SC-classes kept a log of teaching hours and recorded themes of lessons, teaching time using the SC and the method of teaching while using the SC.

In the following, the results of the test are presented along the lines of the six research questions.

RESULTS

Student Achievements (Question 1)

At the start of the school years the SC-classes and control classes took a pre-test. At the end of the school years the post-tests were taken. The diagrams in figs. 1, 2 and 3 compare the results from the post-test for the SC and the control groups.

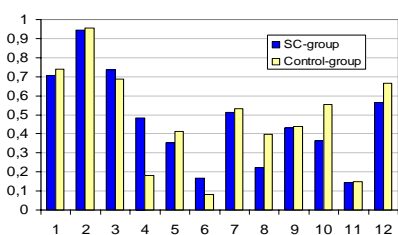


Fig. 1: School year 2003/04.

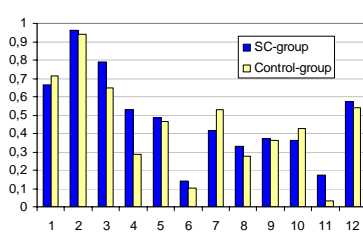


Fig. 2: Year 2004/05.

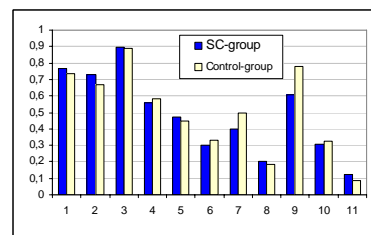


Fig. 3: Year 2005/06.⁴

Test results for the SC and control groups

The results show that the students in the SC-classes achieved better results when working with graphs of functions and doing transfers between equations of

¹ You will find the test questions on www.didaktik.mathematik.uni-wuerzburg.de/weigand/2007/cerme5

² To answer the question whether or how process-oriented competencies have been integrated into the lessons, the lessons had to be (video-taped and) evaluated by a researcher. This will be done in the up-coming phase of the project.

³ See also: www.didaktik.mathematik.uni-wuerzburg.de/weigand/2007/cerme5

⁴ The test questions only changed slightly over the years. Question 12 was cancelled in school year 2005/06.

functions.⁵ No differences between the SC and control classes were observed when working with variables, terms and tables. This discredits the recurring argument that algebraic skills stay underdeveloped with the use of computers. The results did however show a decline in the ability of students in the SC-classes to solve equations of the type $x^2 + 5x = 0$ or $\sin x = 0.5$. The reasons for this are not obvious from the present data.

CAS BENEFITS AND STUDENT DIFFERENCES (QUESTION 2)

In addition, the pre-test results were also used to separate the poorly performing and highly performing groups of students. Based on the total number of points scored on the pre-test, the students were divided into three groups: poor, average and good.⁶ The performance of each of these groups was assessed. (Fig. 4 and Fig. 5).

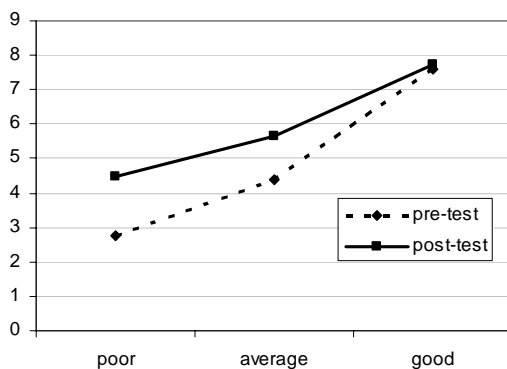


Fig 4: Results of the SC-group for the school year 2004/05.⁷

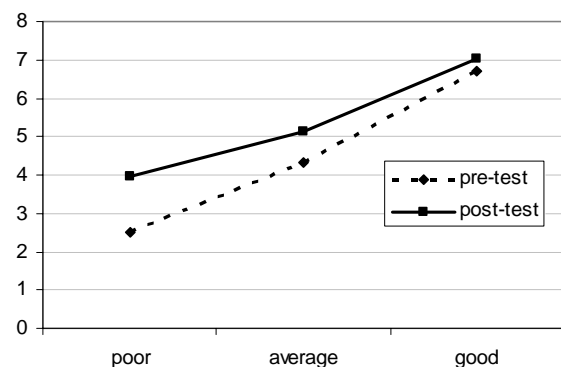


Fig. 5: Results of the control group for the school year 2004/05.

One can see that the “scissors effect”, namely that good students get better and poor students become even poorer, did not occur here. On the contrary, there was an improved performance particularly among the poor and average groups, whilst the ‘good’ students improved only slightly.

There are only hypotheses for explaining these results. Maybe the “good” students were not motivated by the test questions, which presupposed only absolutely necessary basic knowledge. There are a few hints in this direction.

The results also clearly show differences between boys and girls. Standard problems like term manipulations were solved better by boys, whereas “creative problems” like finding formulas from tables were better solved by girls. It is also surprising that the achievement of girls in the control-group increased much more than that of girls in

⁵ The tests were statistically compared with the t-test.

⁶ The average score in the pre-trial test was around 4.5 points. The groups were classified according to the following criteria: poor performing: less than 3.5 points (34 students), average: between 3.5 and 5.5 points (61 students): high performing: more than 5.5 points (30 students).

⁷ We have got nearly the same results for the other school years.

the SC-group. This might be due to the fact that girls may have a negative attitude to the SC and were therefore not sufficiently motivated.

STUDENT BELIEFS AND ATTITUDES (QUESTION 3)

The SC-classes filled out a questionnaire evaluating the lessons with SC. The students were first asked to comment using a five-point scale, and then had to answer three open-ended questions. The following shows the distribution of the answers in percentages:⁸

		++ ⁹	+	O	-	--
1.	The lessons with the Voyage 200 were more interesting than previous lessons.	16 %	40 %	22 %	11 %	10 %
2.	The lessons were easier because of the Voyage 200.	10 %	36 %	22 %	19 %	13 %
3.	The lessons were more varied.	21 %	47 %	14 %	9 %	8 %
4.	I learnt more in these lessons than in other lessons.	2 %	15 %	44 %	17%	22 %
5.	Mathematics in these lessons gave me more pleasure.	8 %	24 %	36 %	15 %	17 %
6.	Using the Voyage 200 has shown me a whole new side to mathematics.	9 %	35 %	22 %	19 %	15 %
7.	I participated more actively than in other lessons.	4 %	11 %	42%	18 %	25 %
8.	I used the Voyage 200 outside of lessons and homework.	8 %	32 %	11 %	25 %	24 %
9.	I'd really like to carry on using the Voyage 200 in mathematics lessons.	32 %	21 %	8 %	11 %	27 %
10.	I would recommend my friends in Year Nine to go into a class that will use the Voyage 200 without hesitation.	14 %	31 %	9 %	24 %	21%
11.	I used the Voyage 200 frequently for my homework.	17 %	42 %	10 %	19 %	11 %
12.	The Voyage 200 is very easy to use.	9 %	30 %	9 %	35 %	16 %
13.	I frequently spent a long time searching for particular commands for the Voyage 200.	15 %	40 %	5 %	32 %	7 %

The answers to questions 8 – 13 clearly divide the group in two.¹⁰ One group liked working with the calculator, used it outside of the lessons, had no great difficulties in

⁸ Values rounded to the nearest whole number. The results are from the school year 2003/04. The results for the school year 2004/05 are quite similar.

⁹ ++: strongly agree; +: agree; o: neither agree nor disagree; -: disagree; -- : strongly disagree.

using it and would like to continue using it. The other group did not enjoy working with the calculator as much as the first group; they worked with the calculator less outside of the course, and had difficulties using the tool.

The mean values for boys and girls differ for all questions, and for almost all the questions these differences are significant. Boys support the use of SC much more than girls - they think they learned more while using the SC and would recommend the use of the SC to their friends.

ASSESSMENT - THE USE OF SC IN CLASS TESTS (QUESTION 4)

Several different research projects have recently looked at the use of CAS in written exams (BROWN 2003). The results largely agree that the structure and type of questions should not be fundamentally changed, and that the CAS students will have a greater variety of solution strategies available to them and will therefore be able to choose their own strategy.

An analysis of the class tests in this project shows that the vast majority of the questions or problems could be presented in the same way in an examination that did not allow the use of a computer. However the use of SC opens up new possibilities for solving problems, since the students have to decide for themselves which method or strategy they will use to solve the given problem. For example, the zeros of a function can be read directly from the graph, obtained by pressing a button (using the “Zero Command”), using a menu command – solve($f(x)=0,x$) – or with the help of a table of values.

COMPETENCES IN SC USE (QUESTION 5)

For bureaucratic reasons, we didn’t have the possibility of evaluating authentic student solutions to problems in class tests. In school year 2005/06 we gave a post-test in the SC-classes that allowed the use of the SC. This test was designed to answer the following questions:

- *How* do the students use the SC?
- Which strategies (symbolic, graphic, and numerical) do they use while solving problems?

Students’ strategies are very much related to how the problem is *formulated*, rather than the problem itself. For example, solving an equation like $\cos\left(\frac{1}{5} \cdot x\right) = x^3$, or determining an intersection point of the graphs of two functions, e. g. $f(x) = \sin(x) + 1$ and $g(x) = 2^x$, are – for the students – different problems. The first is solved with the “solve-command” on the SC, while the second is solved graphically. This shows that the SC is used quite mechanically. The results of the test using SC also show mistakes that were directly related to the handling of the calculator. The calculator is

¹⁰ A more detailed evaluation of the questionnaire revealed that there is indeed a stable group of students who gave answers on the “minus-side” for all questions 8 – 13 and a group of students who marked their answer on the “plus-side”.

not a self-explanatory tool or instrument, and to handle it properly is something one needs to *learn*.

SELF-EVALUATION OF THE TEACHING LESSONS (QUESTION 6)

Lesson protocols

The teacher of the model classes filled in a paper protocol about the use of the SC during the school year, in which the topics covered, amount of time spent, method of teaching, and the predominantly used SC-window were recorded each lesson in different teaching phases.

Content of lesson, exercises, worksheets, etc.	Lesson form while using the SC	Predominantly used SC-window	Length of time in minutes (approx.)...
	Teacher centred Individual work Working in pairs Group work Project work	Algebra Window Graphics Window Table Window Geometry Window	

Record of lessons

The entries were normally filled in after the lesson by the teacher and therefore give only a rough estimation of the timeframe of the use of the SC.

The analysis of the protocols shows that the SC was used in around half of the mathematics lessons. In these lessons, around half the time was allocated to the use of the SC. That does not mean that the computer was used constantly during this time, but rather that the computer was used in these lessons.¹¹

With regards to the use of various teaching styles, we can see that about 30% of teaching included working in pairs or groups, and likewise 30% individual work or student presentations. On the basis of this data, no valid empirical conclusion is possible;¹² however, considering the records of traditional (i.e. calculator-free) German language mathematics lessons (e.g. the TIMSS-Video-Study 1997), do at least establish the hypothesis that the calculator is a catalyst for teaching methods. This has been consistently claimed for German mathematics lessons, e.g. in the current TIMSS and PISA debates.

The teachers reflected on their own teaching in two written reports, one halfway through the year and one at the end. We can therefore summarise their statements as follows:

¹¹ This can mean a demonstration of the computer on the part of the teacher or of a student or the occasional use of the calculator by individual students or also systematic work with the computer by the whole class.

¹² For this an exact record of the timeframe of the corresponding teaching forms of the experiment and of the control class would be required.

- Extended contents: Equations can now be solved which, up until now, had only been dealt with in special cases, for example polynomial equations of higher degrees, exponential or logarithmic equations. Furthermore there is an access to types of functions that had not been considered previously, such as polynomial functions, or discrete logistic growth functions (which could be defined by recursive equations). See Weigand 2004.
- Emphasising flexible thinking: Using dynamic diagrams offers the possibility of working intensively with parameter-dependent functions.
- SC as a control instrument of calculations: The control skills of the students had to be developed, but this was frequently only achieved by the stronger performing students. In contrast, poorly performing students did not manage to successfully search for and remove a mistake when there was a discrepancy between the control and original calculations.
- Discrete operations or activities: Content such as recursively defined sequences gain a greater importance, as there is now a tool available that allows the effective algorithmic handling of mathematical objects.
- All of the teachers were of the opinion that particularly the poorly performing students were very passive when working with the calculator and did not familiarise themselves to any great extent with the use of the tool. For these students the computer remained a *tool* or *artefact* and did not develop into an *instrument*.
- The teachers considered the preparation for the lessons to be considerably more time-consuming because of the inclusion of a new medium, and considered the lessons more challenging particularly as a result of many different tests associated with the content and technical problems. In order to get to know how the lessons and the teaching styles *really* changed, a more extended investigation would be necessary (see. e. g. Kandal a. Stacey 2002).

CONCLUSION

The year-long project shows that the SC was well integrated into the regular teaching of the Year 10 classes. It confirms the following findings:

1. The results of the pre- and post-tests confirm an increase in capabilities in the areas in which an SC can be used as an advantage. Thus, the students of the SC-classes achieved a greater improvement, compared to the control class, in working with graphs of functions and the transfer between equations. Furthermore, no difference was detected working with variables, terms and tables. This shows in particular that algebra skills did not stay underdeveloped with the SC-classes.
2. The improved performance amongst the poor and average students is noticeable. The quite often claimed “scissors effect” (good students get better and poor

students become even poorer) did not occur. But it is very precarious that the best quarter of the students of the SC-classes (concerning to the result of the pre-test) did not improve (concerning to the result of the post-test). The hypothesis for further investigations is that these students were “under-challenged” by the used methods of lessons and that up-coming lessons will have to support these students individually.

3. The beliefs and attitudes of students with regard to the new tools are ambivalent. One group was very willing to use the SC and would like to continue using it in future years. The other group did not enjoy working with the computer, used it less outside of the course and had difficulties using it. For almost all the questions, there are significant differences between the results for boys and girls. There are hints that some girls may have started out with a negative attitude to the SC and, as a result, were not sufficiently motivated and therefore didn't improve their achievement.
4. The questions given on the class tests are not substantially different from traditional class tests. The review of experts of the examination questions, compared to traditional examinations, shows that new solution strategies (e.g. graphical, numerical solutions, experimental methods) are possible, that the computer can be used as a heuristic tool (especially in drawing the graphs of functions by just pressing a button) and as a control instrument to check results obtained with pencil and paper. But these new possibilities have to be learned.
5. Many students – even after one year in class – weren't able to use the computer effectively in new problem situations. The calculator is not a self-explanatory tool or instrument, and one has to learn how to use it.
6. At the least, the lesson protocols of the teachers establish the hypothesis that the calculator is a catalyst for “new” teaching methods. Individual work, as well as partner and group work in mathematics lessons appeared to be reinforced. But teachers claim that the poorly performing students were especially very passive when working with the calculator. Hence, not only the top group in the class but also the weaker students need individual support.

Due to the positive results obtained in this project, the Bavarian Ministry decided to continue the project. The follow-up project was started in September 2005 with 10 classes using the SC in Year 10 and they will continue using this new tool over the next 4 years until their final examination. This will allow a systematic investigation of some open questions identified in this project, and offers the possibility of evaluating the development of long-term competencies.

REFERENCES

ARTIGUE, M., Learning Mathematics in a CAS Environment: The Genesis of a Reflection about Instrumentation and the Dialectics between technical and

conceptual Work, *International Journal of Computers for Mathematical Learning* 7 (2002) 245-274

- BARZEL, B. A. MÖLLER, R., About the Use of the TI-92 for an Open Learning Approach to Power Functions – A Teaching Study, *Zentralblatt für Didaktik der Mathematik* (ZDM) 33 (2001), Bd. 1, 1-5,
- BROWN, R., Computer Algebra Systems and Mathematics Examinations: A comparative Study, *International Journal of Computer Algebra in Mathematics Education* 10 (2003), No. 3, 155-182
- DRIJVERS, P. A. GRAVEMEIJER, K., Computer algebra a an instrument: examples of algebraic schemes, in: Guin, D., Ruthven, K., Trouche, L. (Eds.), *The Didactical Challenge of Symbolic Calculators*, Springer, New York 2005, 163-196
- DRIJVERS, P. A. VAN HERWAARDEN, O., Instrumentation of ICT-tools: The case of algebra in a computer algebra environment, *International Journal of Computer Algebra in Mathematics Education* 7 (2000), 255-275
- GUIN, D., RUTHVEN, K., TROUCHE, L. (Eds.), *The Didactical Challenge of Symbolic Calculators*, Springer, New York 2005
- PESCHEK, W. A. SCHNEIDER, E., Computer algebra systems (CAS) and mathematical communication, *The International Journal of Computer Algebra in Mathematics Education* 9 (2002), 229-242
- PIERCE, R. A. STACEY, K., A Framework for Monitoring Progress and Planning Teaching towards the effective Use of Computer Algebra Systems, *International Journal of Computers for Mathematical Learning* 9 (2004), 59-93
- SCHNEIDER, E., Teacher Experiences with the Use of a CAS in a Mathematics Classroom, *International Journal of Computer Algebra in Mathematics Education* (2000) v. 7(2) 119-141
- TROUCHE, L., Calculators in Mathematics Education: A rapid Evolution of Tools, with differential Effects, in: Guin, D., Ruthven, K., Trouche, L. (Eds.), *The Didactical Challenge of Symbolic Calculators*, Springer, New York 2005, 9-39
- WEIGAND, H.-G., Sequences - Basic Elements for Discrete Mathematics, *Zentralblatt für Didaktik der Mathematik* 36 (2004), Nr. 3, 91-97
- WEIGAND, H.-G. a. WELLER, H., Changes of Working Styles in a Computer Algebra Environment - The Case of Functions, *International Journal of Computers for Mathematical Learning*, Kluwer 6 (2001), 87-111