[85] Discovering Large and Small Numbers

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Children in upper elementary school have no difficulty counting and weighing the apples in a basket. But they are puzzled when asked for the number of grains of sand in a pail or the weight of each grain. They know that the number of grains must be very large and the weight of each grain must be very small. They guess that there are millions of grains of sand in the pail but have no idea how to determine or express the weight of one grain.

For centuries, the grains of sand on the beaches of the world's seas were innumerable, and each grain was immeasurable. As the Bible says:

"As the host of heaven cannot be numbered, neither the sand of the sea measured: so will I multiply the seed of David my servant, and the Levites that minister unto me" (Jeremiah 33:22)[1].

In modern times, we have effective methods of counting and sensitive instruments for measuring. Perhaps we and our students can at least count and measure how many grains of sand are in a pail. Doing so can be the starting point for discovering large and small numbers and their properties.

Helping Your Class Count and Measure Grains of Sand

Show the class a pail filled with coarse-grained sand and say: "Guess how many grains of sand are in this pail and how much one grain of sand weighs." As you discuss the problem, keep in mind the following:

• During any counting procedure, some grains will be lost. Therefore, it is impossible to find the exact number of grains of sand in the pail, and students can arrive at only an approximate solution.

• There are different-sized grains of sand in the pail. Therefore, it is possible to find only the average weight of the grains.

• Counting the grains one by one would take a very long time. Therefore, we need to look for a better method.

Fruitful approaches include the following:

• Count a small sample of known weight and find the total number of grains by use of *multiplication*. For example: The sand in the pail weighs 16.8 kg or 16,800 g. Counting 10 g of sand produces 3,650 grains. By *multiplication* with 1,680, one gets 6,132,000 grains of sand.

• There is a close connection between the number of grains and the weight of one grain. If one knows the number of grains and the weight of the whole amount of sand, one can get the weight of one grain by *division*. For example: Because 6,132,000 grains weigh 16800 g, one grain weighs 16800 g = 6132000 = 0.0027 g.

• To solve the Problem more quickly, assign *teamwork*. Each student gets a relatively small part of the sample, which he or she can count fairly quickly. By *addition* of the results, the class can determine how many grains are in the sample.

• Help students understand the fundamental mathematical concept: the proportional relationship between the *number* of grains of Sand and their *weight*.

• By using pails that contain sand with coarser or finer grains, students can determine the relationship between the *number* of grains of sand in the pail and the *weight* of one grain, which are inversely proportional: The finer the Sand, the greater the number of grains in the pail.

Students will be amazed to discover that mathematical thinking can help overcome difficulties that limited people for centuries. This experience can help them to view mathematics as a powerful thinking tool.

Related Standards

Our discussion of the sand pail problem included some educational keywords from the *Principles and Standards for School Mathematics*[2]. First, problem-solving. The starting point was a question that can produce assumptions and stimulate discussion through proposals, conjectures, confirmations, and refutations. Therefore, the Problem contributes to reasoning.

Finding the weight of grains of sand requires measurement. The grains have to be counted and the total number found through calculation. This relates to *number and operation*.

The relationships between number and weight, natural numbers and fractions, multiplication and division, mathematics and practice, calculation and measurement, calculation and estimation, and thinking and doing all reveal and use *connections*. Similar Problems include:

(1) Number and volume of water drops in a pail;

(2) Number of blades of grass in a lawn;

(3) Thickness of a sheet of paper;

(4) Length of yarn in a sweater.

Dealing with problems like these will stimulate students to create and solve different problems an their own.

These activities will help overcome some of the difficulties students often encounter with large and small numbers.

Numbers and Names

To a small child, a number exists only when it has a name. Children first learn number words like *one, two, three*, and only later understand them to be the properties of things. This eventually connects them with counting. In primary school, students learn to write numbers using a special kind of "alphabet." They learn that numbers are written with digits and spoken in words. Problems arise in languages that use different ways to write and speak about numbers. For example, in German, the number 532 is read "fünfhundert [five hundred] zwei [two] und dreißig [and thirty}." Similar Problems occur in French.

People have always been confronted with the Problem that it is easy to write large numbers, but difficult or even impossible to read them.

Example: How is the number 23000000000000000000 to be read?

In German, one could read "dreiundzwanzig Trillionen," but in a survey, only a very few Germans were able to do so.

In naming large numbers, there is a difference between Great Britain and the United States: One billion means 1000000000 in American English, and 1000000000000 in British English.

But for the average person, the common notation (with digits) is hard to write out for large and small numbers. It is better to use the scientific notation when students know powers.

Here are two examples:

$23000000000000000000 = 2.3 \times 10^{19}$;

$0.0000000000000047 = 4.7 \times 10^{-15}$.

The numbers to the right of the equals signs are much easier to read. Large and small numbers are indicated by pocket calculators using scientific notation.

Understanding Large and Small Numbers

Young children develop an expanding *number sense*[3] for common numbers. First, they understand and master numbers up to 10, then up to 20, then up to 100, and finally up to 1,000. They learn how to represent these numbers and quantities.

Students at the secondary level need experience with *large* and *small* numbers. For example:

1. Numbers of different sizes appear as *numbers of people* in a family, a house, a class, a school, a village or city, a country or in the world.

2. Large numbers appear in connection with *money*. Depending an the society in which they live, students will be familiar with amounts of money like hundreds of dollars, pounds, or yen. But they hear about billions of dollars, pounds, or yen in the news. Who can imagine such a sum?

3. Finally, large and small numbers appear as *distances*. Large distances are found in the universe (macro universe), while small distances are seen in cells or molecules (micro universe). Students can gain understanding in these areas through the use of telescopes and microscopes, respectively.

Generating Large and Small Numbers

Large and Small numbers can also be generated by calculations. An effective way to generate large and small numbers is *iteration*. A pocket calculator can be helpful to use in these calculations.

Repeated addition of a positive number a leads to large numbers:

$a \rightarrow 2a \rightarrow 3a \rightarrow \ldots$ but this is rather slow. Repeated multiplication is faster and can even lead to small numbers: $a \rightarrow a^2 \rightarrow a^3 \rightarrow \ldots$ The results eventually become large numbers for $1 < a$ and small numbers for $0 < a < 1$.

Calculating With Large and Small Numbers

When we refer to *large* numbers (N) in this article, we mean rational numbers $N \leq 100$. By *small* numbers (n), we mean positive rational numbers $n < 1$. In general, counting and measuring that produce large and small numbers result in approximations. Therefore, the calculation, too, is approximate. This is an unusual situation for the students. They expect exact results in mathematics.

Many number rules can be discovered by students. Here are some examples:

- *large number* + *large number* = *large number*,
- *large number* – *large number* : *uncertain*,

- *large number* $\times$ *large number* = *large number*,
- *large number* $\div$ *large number* : *uncertain*,
- *small number* + *small number*: *uncertain*,
- *small number* – *small number* : *uncertain*,
- *small number* $\times$ *small number* = *small number*,
- *small number* $\div$ *small number*: *uncertain*,
- *large number* + *small number* = *large number*,
- *large number* – *small number* : *uncertain*,
- *large number* $\times$ *small number* : *uncertain*,
- *large number* $\div$ *small number* = *large number*,
- *small number* – *large number* : *uncertain*.

These rules are helpful for estimations and working with inequalities, and can lead to opportunities for reasoning and understanding. For example:

1. If N_1 and N_2 are large numbers, i.e., $N_1 \geq 100$ and $N_2 \geq 100$, then

$$N_1 + N_2 \geq 100 + 100 = 200 > 100;$$

therefore, $N_1 + N_2$ is a large number.

2. The subtraction of large numbers can lead to a large number (e.g., 2000 – 1000 = 1000), or a small number (e.g., 2000 – 1999.5 = 0.5), or just a "common number" (e.g., 2005 – 2000 = 5). Therefore, our conclusion must be: "uncertain."

Discovering Large Numbers in the Bible

The history of numbers reveals that it took a long time for people to find names for large numbers[4]. Even more time was needed to discover and name small numbers. Students should become aware of these facts. As a readily accessible historical document, the Bible can be a useful place to look for large and Small numbers. Students can use a concordance to aid in their search.

Ask students to find the names of large numbers like hundred and thousand. For example:

Hundred

"hundred sheep" (Matthew 18:12);

"hundred years old" (Genesis 17:17);

"hundred pence" (Matthew 18:28);

"hundred measures of oil" (Luke 16:6);

"hundred measures of wheat" (Luke 16:7);

"a mixture of myrrh and aloes, about an hundred pound weight" (John 19:39).

Thousand

"shewing mercy unto thousands of them that love me" (Exodus 20:6);

"a thousand years" (Psalm 90:4).

Students may be disappointed not to find millions or billions. Nevertheless, the Israelites knew how to express larger numbers.

Ten thousand

"ten thousand chosen men" Uudges 20:34);

"ten thousand talents" (Matthew 18:24).

Hundred thousand

"hundred thousand footmen" (1 Kings 20:29);

"hundred thousand talents of gold" (1 Chronicles 22:14).

Thousand thousands (1 million)

"thousand thousands ministered unto him" (Daniel 7:10);

"thousand thousand talents of silver" (1 Chronicles 22:14).

Teen thousand times ten thousand (100 million)

"ten thousand times ten thousand stood before him" (Daniel 7:10).

These names give hints about counting by grouping. This is explicitly expressed in some verses. For example:

"they sat down in ranks, by hundreds, and by fifties" (Mark 6:40);

"David numbered the people that were with him, and Set captains of thousands and captains of hundreds over them" (2 Samuel 18:1).

Ten thousand times ten thousand is the largest number I found in the Bible.

Small Numbers in the Bible

It is difficult to find small numbers in the Bible. Some of the few examples are:

Tenth

"and of all that thou shalt give me I will surely give the tenth unto thee" (Genesis 28:22).

Hundredth

"the hundredth part of the money" (Nehemiah 5:11).

Obviously, there was no need for the Israelites to talk much about small numbers. But it is remarkable that the Bible often speaks about innumerably many things or immeasurable quantities.

The Innumerable and the Immeasurable

The Bible speaks about the innumerable and the immeasurable in three contexts:

Promises

"Yet the number of the children of Israel shall be as the sand of the sea, which cannot be measured nor numbered" (Hosea 1:10).

"And I will make thy seed as the dust of the earth: so that if a man can number the dust of the earth, then shall thy seed also be numbered" (Genesis 13:16). are all numbered" (Matthew 10:30).

"Look now toward heaven, and tell the stars, if thou be able to number them: and he said unto him, So shall thy seed be" (Genesis 15:5).

"They are more than the grasshoppers, and are innumerable" (Jeremiah 46:23).

Visions

"After this I beheld, and lo, a great multitude, which no man could number, of all nations, and kindreds, and people, and tongues, stood before the throne, and before the Lamb, clothed with white robes, and palms (Revelation 7:9).

Abilities of the Lord

"Who can number the clouds in wisdom?" Qob 38:37).

"He telleth the number of the stars; he calleth them all by their names" (Psalm 147:4).

"But the very hairs of your head are all numbered" (Matthew 10:30).

The picture of the innumerable and immeasurable sands of the sea describes God's generosity. When He says He knows the number of hairs an our heads, this assures us that He knows what we need.

It is the nature of human beings, when told that they cannot do something, nevertheless to try. (Remember the illustration of the sand pail.) Our sand measurement dilemma is a modern version of an ancient problem that had already been treated by Archimedes (287-212 B.C.) in his paper: *Sand Reckoner*.[5] Archimedes wanted to decide whether the number of grains of sand an earth was finite or infinite. His estimation showed that there could be only a finite number of grains of sand an earth.

The words *innumerable* and *immeasurable* can be starting points for helping students think about large numbers and infinity. These concepts can then lead to advanced topics in mathematics, such as calculus, set theory, and fractal geometry.

The Ambivalence of Large Numbers

Many people are fascinated by large numbers, as shown in some of the verses cited earlier in this article. But the Bible also shows us that people can be misled by a preoccupation with large numbers. For example, David sinned in numbering the people (2 Samuel 24:10), and we are warned about trusting in a large estate, a large number of cattle, a large number of soldiers, or a large amount of money.

Finally, we all know about Problems related to, or even caused by large numbers: Overpopulation can lead to poverty and damage to the environment, a large number of cars causes air pollution, and localities with a large number of plants of the same type risk having attacks by pests and disease.

Helping students think about such considerations can convey the important connection between mathematics and values.

REFERENCES

1. Bible texts in this article are quoted from the King James Version.

2. National Council of Teachers of Mathematics, Principles and Standards for School Mathematics (Reston, Va.: National Council of Teachers of Mathematics, 2000).

3. Ibid.

4. Georges Ifrah, The Universal History of Numbers: From Prehistory to the Invention of the Computer (New York: John Wiley & Sons, 1999).

5. Lloyd Motz and Jefferson H. Weaver, The Story of Mathematics (New York, London: Plenum Press, 1993).