66. Discovering Geometry Through Patch-Pictures

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When German fourth graders see the patch-picture above, most of them describe a town with a church building, houses, a bus, and trees. The same would be true in many other parts of the world. But what would African or Eskimo children say? Perhaps these students would see only a collection of fancy patches. Suppose I told you that these are just a set of geometrical shapes? You would certainly feel cheated. Normally we are so impressed by a picture that we fail to see its elements. Let's analyze the picture: It consists of rectangles, squares, triangles, circles, and other shapes. The objects are combinations of these shapes. The church consists of rectangles, a triangle, a circle, and a trapezoid. The bus is formed from rectangles and circles. If we look further, we discover overlapping shapes:

- The windows are smaller rectangles placed on a large rectangle.
- Two buildings overlap, giving the impression that the house is behind the church.

How does this relate to geometry? Students begin to do geometry when they discover:

1. There are *prime* figures such as squares, rectangles, triangles, and circles.

2. When such figures are combined, *new figures* can be *generated,* such as parallelograms, trapezoids, etc.

3. There are many kinds of relations between figures in the plane. Figures can be separate or they can overlap; their relation to each other can be described in positional terms Tike right, left, above, below, etc.

1. Knowing figures

Students usually learn the names of the prime shapes in primary school. When you ask them, "Why is this figure a square?" they will tell you, "Well, that's what I see." This seems like a rather inadequate explanation. But according to van Hiele, it is the first stage of understanding a geometrical figure: the figure as a whole is seen as a certain form (VAN HIELE 1986). In psychology, this is known as "gestalt."

You can test your students' understanding of this stage by asking them to name a variety of paper shapes. (See Figure 1.) Or you can ask them to choose a certain shape from a pile of figures, e.g., all the triangles.

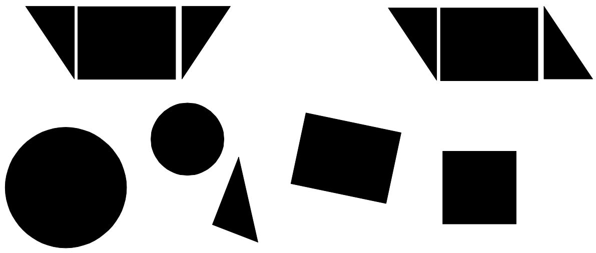

Fig. 1

This stage of understanding is often underestimated in geometry teaching. Many teachers hurry past it to drawing polygons and looking for properties such as equal length of sides, equal angles, etc.

This is the second stage of understanding: seeing figures as carriers of properties. But don’t skip over the first stage because there are some difficulties to overcome. First, ask your students to name the shape in Figure 2.

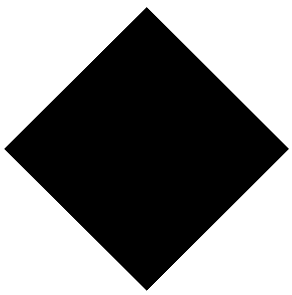

Fig. 2

If they feel lost, or call it a diamond or rhombus, this means that they cannot identify a figure independent of its orientation in the plane. They can learn to do so if you present the shape in many different situations.

When students see a complicated figure for which they do not know a geometrical name, they will describe it in terms of objects in their environment. For instance, in Figure 3, the shape on the left may be called a "bridge", while the shape on the right may be described as a "groove".

Fig.3

One could describe this as a figure that resulted from cutting off a semicircle from a rectangle, but in most cases, the environmental interpretation dominates.

2. Geometry and Reality

It is easy to discover geometrical shapes and their relationships from a patch-picture. But this is just a picture. Can we also teach students to discover these shapes in real life? Can one really see a church, a bus, or a tree the same way it appears in the picture? One answer could be, "Yes, when the distance between them and the object is large enough." Another answer could be, "Yes, by looking at the shadows of these objects."

On the other hand, we have to neglect or ignore a lot of properties to discover geometry in our environment. For example, color, material, and finish all create distractions. We need an abstraction from all these properties.

But, in Figure 4, if you compare the photo of the car and the patch-picture, obviously some things have been omitted and others have been changed.

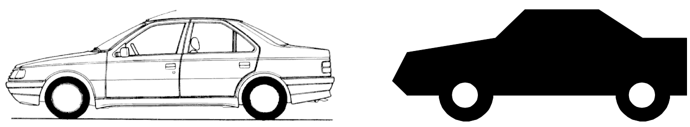

Fig. 4

Take for instance the boundary line. Here we have an interpretation: the real boundary line has been transformed into a simpler line. An idealization has taken place. Abstraction and idealization are fundamental cognitive processes that lead from reality to geometry. When we use these processes, we see the world geometrically (VOLLRATH 1976). Understanding the world in a geometrical way helps us to find our way around.0

3. Communicating Geometrically

Presenting a picture, interpreting it, and exchanging pictures can be a way of communicating. Peter makes the patch-picture in Figure 5 and asks his neighbor Anne, "What is this?"

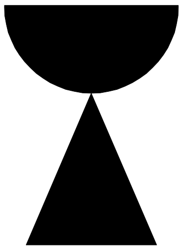

Fig. 5

Anne answers, "It’s an egg-cup!" Peter is happy because Anne understands his picture. But Anne might say that the figure is a drinking glass. Peter then would feel misunderstood.

Because of its universality, geometrical language is ambiguous when we apply it to reality. But often the context helps us to better understand (VOLLRATH 1977). Pictures can help children who speak different languages to communicate geometrically.

When students see a patch-picture, they will want to make some themselves. Sturdy colored paper (with or without sticky backing) works well. The shapes can be cut out freehand. However, it is easier to draw the shapes with stencils, which can be made at the beginning of class. By cutting off parts anti combining pieces, students can create the shapes they have in mind.

Solving problems often leads to new discoveries. For instance, the students may want equal-sized windows. They can use the same stencil over and over. Or they can put several shapes on top of one another and then cut them out.

Students thus discover the principles of congruent figures. Or they may want to make a fir tree that looks the same on both sides. They fold the paper and cut out the half tree. They have discovered reflectional symmetry.

Use of ruler, protractor, and a pair of compasses would be premature at this stage. You can teach about figures as "carriers of properties" at a more advanced stage.

4. Being creative in Geometry

Mathematics is not often thought of as promoting creativity. Working with patch-pictures can change this perception. As children arrange different shapes, form complex shapes, test their impressions, change the shapes, and again test the result, they will gain experience and become familiar with geometrical shapes. You can encourage them by showing an interest in their creativity, talking about the meaning of certain arrangements, and suggesting changes.

Patch-pictures can be a valuable means to teach geometry, particularly for slower anti less motivated students.

5. Cooperating Geometrically

Working cooperatively to make a patch-picture is an ideal projeet to advance students' knowledge of geometry. "Our city," "our village", "our church", or "our camp" can be themes (JOHNSON, JOHNSON 1987). One group can form houses, while other groups make trees, animals, flowers, cars, and people. The students can work together on the final picture, which can be displayed in the classroom for parents and friends to admire. The children will have a wonderful experience as they contribute to the whole. As they admire their work, you can explain that mathematicians, too, cooperate in economy and industry to organize projects that can be performed by the computer.

In conclusion, making a patch-picture can be an early and important mathematical and social experience for your students. Why not try it?

References

Hiele, P. M. van, Structure and Insight, Orlando (Academic Press) 1986.

Johnsan D. W., R. T. Johnson, Learning Together and Alone: Cooperative, Competitive, and Indlividualistic Learning, Englewaod Cliffs (Prentice-Hall) 1987.

Vollrath, H.-J., "The Place of Geometry in Mathematics Teaching", Educational Studies in Mathematics 7 (1976), .431-442.

Vollrath, H.-J., "The Understanding af Similarity and Shape in Classifying Tasks," Educational Studies in Mathematies 8 (1977), 211-224.