35. A case study in the development of algebra teaching in the Federal Republic of Germany

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During the last 20 years there has been a rather important change in algebra teaching in the Federal Republic of Germany. I want to show the origin of this reformation, the underlying ideas, its main phases, and some of the mechanism in the development. For the reasons of a case study it seems to be convenient to restrict the considerations to a special theme of algebra teaching. A very important point of discussion has been the "theory of equations" (Gleichungslehre). It seems to be fruitful to take this as a case study for curriculum development, because it was a specific German development, in which typical processes might be seen, its motives, its results and its progress. Curriculum changes in a special subject of mathematics instruction are very often embedded in wider processes of curriculum development. Sometimes it happens that reformers are not aware that they are walking on moving ground. The process of developing the theory of education during the last 20 years is embedded in the general curriculum reform movement in mathematics that took place in these years. I have chosen to study the main phases of the development of this special theme because it is easier to identify the determining factors in a restricted field.

Phase 1: Release

In 1960 H.-G. STEINER read a paper at the Düsseldorf conference of the MNU (Verein zur Förderung des mathematischen und naturwissenschaftlichen Unterrichts) on logical problems in mathematics instruction, and produced some new ideas for teaching equations. His ideas were influenced by TARSKI's book on mathematical logic (l941).At that time STEINER worked at the "Seminar für Didaktik" in Münster where fruitful discussions among mathematicians and mathematics teachers took place. Therefore STEINER had a good basis for his ideas. The main response was a very well grounded paper from WÄSCHE (teacher at a Gymnasium in Lübeck) in 1961 on logical problems in the theory of equations and inequalities. This phase is characterized by criticism and suggestions.

i The traditional handling of equations in algebra teaching was full of conceptual weaknesses. There was a pseudo-systematic hierarchy for equations: " identical number equations" (e.g. 2x3=5), "identical letter equations" (e.g. ab+ab = 2ab), "formulas", "defining equations", "determining equations", "function-equations" etc. It was suggested that this terminology would be unified if one sets the problem within the context of logic and set theory. A special emphasis was put on the understanding of the variable as a "place holder" or as the name of an "empty place”.

ii Transformations for equations had been mainly used mechanically ("If you put a number on the other side change the sign"). The students only had a very vague idea about the reasons for the transformations. Much emphasis was therefore put on founding the transformation rules (by rules of numbers) and to interpret the types of transformation ("profit-transformations", "deficit-transformations", and "equivalence-transformations").

iii In the traditional teaching of equations the role of the underlying set had been neglected. It was an important aim to make clear the influence of the ground set on the solution set. One expected better understanding of the different number sets.

iv Special cases such as unsolvable equations or universally valid equations were neglected or not discussed under the aspect of solving equations. The result was that unsolvable equations were considered as pathological, whereas universally valid equations were considered as algebraic laws and not under the theme "equations". Therefore the suggestion was made to include these types among problem sequences of equations.

v Inequalities did not play an important role in traditional algebra teaching. This was a handicap for the teaching of calculus. Therefore the suggestion was made to teach inequalities together with equations, as "statement forms" (open sentences). One expected a better understanding of the transformation rules when discussing analogies and contrasts. A further argument was the possibility of getting solution sets with more than one solution in an early stage of teaching.

Without any doubt most of these ideas were accepted by progressive teachers. Some reasons might have been:

i predilection for precise language,

ii stimulation for theory forming,

iii fascination by modern mathematics (especially logic, sets, and structures),

iv need for improvement of student's understanding and skills,

v preference for algebra in mathematics instruction,

vi general curriculum reform.

The main assurance was that by an innovation in this direction the algebra teaching could be improved ("Student Outcomes"). But on the other hand personal preferences of the teachers might have had some influence too.

Phase 2: Methodological preparation

Stimulated by the papers of STEINER and WÄSCHE a great variety of papers, project materials, teacher training materials, and textbooks were developed, each of them trying to solve methodological problems (or just to copy).

i It was necessary to work out the theme into details. In this the papers of LAUTER (at that time teacher at a Gymnasium in Aachen) and WÄSCHE 1964 were a great step forward. They developed the framework for the curricula in textbooks. (And with hindsight it is also possible to see the roots of some misdevelopments).

ii Some conceptual problems arose when the theory was worked out in detail. E.g. the role of undefined terms (e.g. $1 - x$; $\sqrt{\frac{1}{x-1}}$ etc.) was discussed. The suggestions ranged from very sophisticated ideas (MARKWALD) to very simple solutions (PICKERT).

iii The theory of equations had to be embedded into the algebra course to permit the development of the number systems. The main problem was how to integrate the modern theory of equations in a natural way. Some criticisms made against textbooks were that the modern concepts were grafted upon traditional content (e.g. against REIDT-WOLFF-ATHEN 1964) . Equation theory should be formed fundamentally from new.

iv The suggestion was made to start solving equations with exercises of "systematic placing in". The aim was to avoid early mechanical techniques of solving. To derive the set of solutions it was therefore necessary to bound the ground set. Therefore finite sets were used (and misused). This was extremely treated in KUYPERS' textbook 1964 (influenced by LAUTER). But the origin of this idea is STEINER's proposal to stress the "semantic" aspect of equations.

v Solving equations and inequalities was understood as theory that should be developed in mathematics instruction. Therefore extensive texts were given to inform students (and teachers) about the underlying ideas, the use of the concepts and the foundations of the rules and techniques (SCHRÖDER-UCHTMANN 1967). This was based on the tradition in the Gymnasium to offer descriptive rules for handling mathematical subjects. The teachers were won over to such ideas by the modern textbooks by teacher training materials (e.g. GRIESEL, HÄNKE) and in inservice training held by "Fachleiter" (heads of department), textbook-writers and experts in mathematics education from universities and teacher training colleges. These persons also tried to introduce these ideas into the syllabuses worked out in each of the "Länder". In particular these efforts ("Control System") must be reviewed as part of a more general curriculum revision in Germany (DAMEROW).

Phase 3 : Expansion

Most of the work done in the first two phases was directed at Gymnasium students, grades 7-70 (i.e. l2-16 age-range). By 1970 these ideas extended to influence earlier ages and school-types.

i It is a principle of the Gymnasium to teach mathematics in the lower grades bearing in mind the needs of the upper grades. Therefore each subject must have its "Propädeutik". Consequently suggestions were made to start with equations already in grade 5. One did equations without transformations and developed the conceptual apparatus for very simple cases as an application of set theory.

ii In the primary school equations placeholders like ■ and ● were introduced to let students work with numbers in an operational way (PIAGET, FRICKE-BESUDEN, BECKER).

iii To offer equal chances to all students, it was felt that students at the Hauptschule should also learn mathematics and not only arithmetic ("Rechnen"). Therefore these students also were offered equations. (It was an early suggestion e.g. by BREIDENBACH, to deal with equations in the Hauptschule, but it was only seldom practised). And this was also done with all the conceptual framework (HOFSÄSS, OEHL, GRIESEL-SPROCKHOFF), which had been developed for the purpose of the Gymnasium.

These efforts were successfully translated into syllabuses and textbooks, because of the influence of the mathematics educators at the universities and at the teacher training colleges in their teaching of future teachers, at inservice training courses and in the various syllabus committees, and also because of the interest groups in the schools (supervisors, teacher unions, teachers) for whom it often was a question of status to teach modern mathematics more theoretically. And again one must also take into account that this process was embedded in a general development of the curriculum and also in the discussions about a modernschool system (DAMEROW).

Phase 4: Reduction

During the last 5 years a lot of criticism has arisen against the way equations are taught in practice.

i There has been a strong criticism against the excess of formalism (PICKERT, VOLLRATH, BARTH). It was seen, that e.g. the calculus of term transformations was unnecessary (PICKERT). There should be no “ground set acrobatics". It can be avoided by using the maximal known number system as ground set (VOLLRATH) .

ii The influence of formal logic was felt to be too strong. Therefore suggestions were made to make more use of the conceptual framework of functions (VOLLRATH, BRÜNING-SPALLEK). Equations can arise from questions about functions, and transformations of equations can be interpreted as transformations of functions.

iii The extensive explanations given in textbooks were felt to be too strongly teacher-oriented. New text books try to avoid such extensive descriptions (e.g. HAYEN-VOLLRATH-WEIDIG).

iv Initially, instead of "systematic testing", simple methods of reasoning are used. An important aid has been the use of operators: e.g. 2x+3=11 is solved by

$$x \xrightarrow{\cdot 2} 2x \xrightarrow{+3} 2x+3$$

$$4 \xleftarrow[:2]{} 8 \xleftarrow[-3]{} 11$$

Here again the idea of function has become an aid.

v Suggestions have been made for avoiding unnecessary formalism in the Hauptschule and for developing a better understanding and better skills in working with formulas, which are very important in the future commercial life of the students (VOLLRATH, WINTER).

The emphasis of this phase is directed more to the needs of the students. Criticism is mainly made against formalism and conceptual pomp. On the other hand this coincides with a stream of restoration which tries to lead back to the past. Most of the changes in this phase are part of a "silent reform" of textbooks. The reactions of teachers vary. Many of them, especially in the Gymnasium, prefer the more theoretical way of teaching equations with a large conceptual apparatus. More student-oriented teachers seem to prefer the latest approaches. Especially in the Hauptschule the latest programs are accepted because they are felt to take better account of the limited abilities of the students ("Implementation")

Evaluation:

As a result of 20 years of developing a theory of equations for students I would presume:

More students know more about equations.
More students can solve equations and inequalities.
More students can speak about equations.

But I also would fear:

Many students do not like equations.
More students do not like inequalities.
Not enough students can handle equations and inequalities correctly.

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