REASONING WITH MULTIPLE AND DYNAMIC REPRESENTATIONS

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This paper presents a research in progress which aims to answer the question if learners working with digital multiple and dynamic representations really refer to the dynamics and the multiplicity in their arguments. In an empirical investigation learners were given mathematical reasoning tasks along with the appropriate representations, which were partially multiple, dynamical, or both. The hand-written documents were analysed regarding the appearance of multiple or dynamic representations in the learners arguments. Results indicate that the given representations have a great influence most of the time, but not always.

MULTIPLE AND DYNAMIC REPRESENTATIONS

Representations play an important role in the learning and the use of mathematics. The emergence of new technologies such as computers and handheld devices, the quick availability of even complex representations, along with the possibility of creating multiple and dynamic representations of mathematical objects with the press of a button, brought their use in mathematics classrooms into research focus.

Using multiple representations yields potential benefits for learners. As Ainsworth (1999) pointed out in her taxonomy of the roles of multiple representations, they can not only provide learners with additional information and allow them to use more appropriate strategies for their task, but they can also help to decrease ambiguity and construct a deeper understanding of representations. We speak of a multiple representation (MER) if it integrates and displays more than one representation of the same mathematical object. Otherwise we call it isolated (IER). An example of a multiple representation is the simultaneous depiction of an equation, a graph and a table of a function.

Dynamic representations (DER), too, provide learners with additional information by enabling them to vary a given representation quickly to construct variations of a mathematical problem which may lead to conjectures about a solution (Arzarello, Ferrara, & Robutti, 2012). We will call a representation dynamic if it changes over time (automatically or through user input). Otherwise, we call it static (SER). As an example of a dynamic representation one may think of using sliders to vary parameters of an equation of a function resulting in a change of the shape of the function's graph.

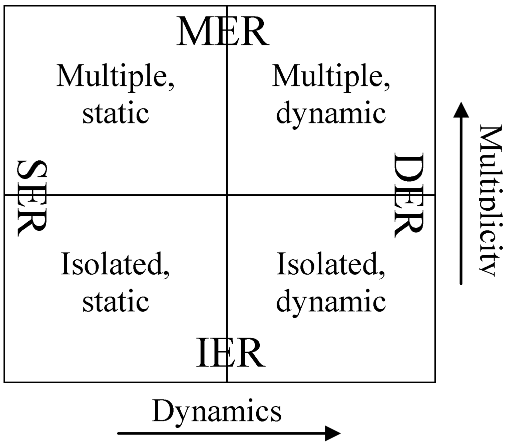

Fig. 1: Taxonomy of representations

RESEARCH QUESTION

The presented research was designed to answer the following question:

Does the representation category (s. figure 1) that is used when presenting the problem to learners have the most significant impact on the representations used by learners in their justifications, or is it their individual preference (or capability)?

METHOD

The empirical investigation was taken in four German classes, grade 11 (students aged 16-17 years); a total of 89 probands with mixed levels in school mathematics. Their tasks were presented on a computer screen, with the students being in control of the input devices to enable them to vary the given representations if possible. The students were asked to solve the tasks with paper and pen, so they would not be limited by having to give mathematical input to a computer. The probands were divided into two groups (sized 44 and 45 students) to ensure that while all combinations of representational category switching were present, the working time did not exceed 45 minutes.

The tasks have been around the mathematical topic "functions". The questions were purely inner-mathematical, and students were always asked to give justifications for their claims. Each student had to solve four mathematical problems, which consisted of two pairs a and b with analogue tasks, but different representations, in an "abab" order (see below in the results). This design was used to determine if the students were set on one category of representation as in figure 1, regardless of the one presented, or if they switched e.g. from using isolated to multiple representations, according to the one presented in the problem.

RESULTS

Not all probands completed all tasks. This is why the following numbers do not add up to 89. The p values were calculated using the binomial test where the null hypothesis is that the considered types of representations in students' arguments were distributed equally.

Table 1: Results of group A

		A3 (IDR)				A4 (MDR)	
		Stat. Arg.	Dyn. Arg.			Isol. Arg.	Mult. Arg.
A1 (ISR)	Stat. Arg.	1	**9**	A2 (IDR)	Isol. Arg.	2	**14**
	Dyn. Arg.	2	28		Mult. Arg.	2	10

As the table above shows, 9 students who used SER in their arguments switched to DER when the representation depicted in the task switched in the same way from task 1 to 3. Although the majority of 28 students used DER in both cases, this is a significant result ($p < .05$). In tasks 2 and 4, when multiplicity was added, but the representation remained static, the largest group of students (14) switched from isolated to multiple representations, which is a highly significant result ($p < .01$).

Table 2: Results of group B

		B3 (MSR)				B4 (MDR)	
		Isol. Arg.	Mult. Arg.			Stat. Arg.	Dyn. Arg.
B1 (ISR)	Isol. Arg.	23	**16**	B2 (MSR)	Stat. Arg.	14	5
	Mult. Arg.	2	1		Dyn. Arg.	11	9

While 23 students stuck to isolated representations in task 1 and 3, a large group of 16 switched to MER in their arguments, also a highly significant result ($p < .001$). The switch from multiple static to multiple dynamic representations, however, did not show a clear picture, resulting in $p > .10$. The reason for this is not clear and suggests further investigation.

REFERENCES

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